DISPERSION IN VERY BUOYANT PLUMES

D M Webber
Integral Science and Software Ltd, 484 Warrington Rd, Culcheth, Warrington, WA3 5RA

S J Jones and G A Tickle
AEA Technology plc, Thomson House, Warrington Rd, Risley, Warrington, WA3 6AT

Very buoyant plumes which may arise in large fires may disperse material differently from slightly buoyant, Boussinesq plumes. This paper reports on one aspect of a joint European project of wider scope. In particular, here, we focus on modelling turbulent, isothermal, plumes of high buoyancy, in a calm, unstratified atmosphere. The model has much of the complex structure expected in very buoyant plumes, compares well with laboratory data, and, although one-dimensional, is shown to be consistent with the full three dimensional turbulent fluid flow equations. It therefore provides a sound scientific basis for further development of models of "more realistic" situations, and an excellent analytic verification test in the isothermal limit of such models. In this way it forms a concrete illustration of the general principles of sound model development expounded by the EC Model Evaluation Group.

Keywords: plume, buoyant, calm air

INTRODUCTION

We will report here on part of project supported by the European Commission to investigate the behaviour of very buoyant plumes. Such plumes may arise from large fires. In modelling these the Boussinesq* approximation, frequently used in other cases, will be invalid in most or all of the region of interest. The overall project aims to study buoyant plumes from the source up to a possible inversion high in the atmosphere, in different atmospheric conditions. Here we shall restrict attention to the earliest part of the project: the study of very buoyant isothermal plumes, close to the source, in calm air.

At this stage it therefore falls short of a complete usable model for hazard analysis, but it is also one of the objectives of this presentation to provide a more general blueprint of how such models are constructed. In recent years there has been a good deal of concern over the question of fitness-for-purpose of hazard analysis models. Such models are used to predict events which we hope will never happen, and which are too dangerous to permit full-scale direct experiments. How do you tell a good model from a bad one? If you believe you have a good model, how do you prove that to the world at large?

The EC brought together a group of scientists and engineers, the EC Model Evaluation Group, to study just these questions. It has published a pamphlet (ECMEG 1) giving the general principles

* If the density of the plume is very close to that of the ambient air, then the effect of variations in density on inertia may be ignored. The density variations must only be accounted for in the buoyancy driving force, where it is the density difference which is crucial. This is known as the Boussinesq approximation. We do not adopt it here.

which must be followed if one has to confidence in a mathematical/computer model for hazard analysis. There are essentially three principles: (i) the model must have a sound scientific basis; (ii) any computer code must be shown to be solving the equations of the model accurately (irrespective of their validity); and (iii) the model must be shown to fit a wide enough set of experimental data (including data which have not been used to tune any free parameters).

One can find models which violate any or all of these principles, but (i) is perhaps arguably the most frequently violated. But, for example, it is clearly insufficient to have a well constructed computer code which fits small scale experimental data if one has no reason to believe that it has any validity when used to predict the behaviour of much larger occurrences which may be the result of real accidents. Whether or not the model is scientifically sound is of course a matter of scientific judgement, on which traditionally a consensus is formed after publication in a peer reviewed journal or conference proceedings. In the case presented here, in deriving the buoyant plume model we are asking whether models applied to slightly buoyant flows can be validly applied to very buoyant flows, or whether we need something better. We have explicitly investigated the scientific basis of our isothermal model.

However questions about intense thermal effects remain to be answered - but that is beyond the scope of this work. For the moment we prefer to turn this about, and note that any model of a thermal plume should be soluble in the isothermal limit. And in this limit it should reduce to something like the simple analytic plume model given below. Thus the ideas presented here may be used to provide a check on the accuracy of a more complex computer code, according to principle (ii) above.

Finally, following (iii) above, we shall show a comparison of the model with isothermal buoyant plume data, tuning two or three free parameters to a data set comprising many more points. We therefore hope that the model presented here will serve as a concrete illustration of the processes recommended by the EC Model Evaluation Group.

A MODEL OF A RISING PLUME IN CALM AIR

The model

Simple one-dimensional models of rising plumes are straightforward to construct and we shall give one below. First let us note that one-dimensional models of an isothermal plume give rise to a conserved buoyancy flux of the form

$$B = \frac{(\rho_a - \rho)}{\rho_a} \pi R^2 w \tag{1}$$

where ρ,R,w are the plume density, radius, and upward velocity of the plume (we'll discuss more precise definitions briefly later) and ρ_a is the ambient air density. The two dimensioned parameters B and g (the acceleration due to gravity) can now be combined to give unique length and velocity scales (L,U) for the physical processes in the plume conveniently defined as

$$L \equiv \left[\frac{B^2}{\pi^2 g}\right]^{1/5} \qquad U \equiv \left[\frac{B g}{\pi}\right]^{1/5} \tag{2}$$

These and the density scale ρ_a allow us render all the variables in the problem dimensionless, and we shall consider dimensionless variables only from this point to the end of the section. In dimensionless terms, our model of the plume is

$$\rho = 1 - c \tag{3}$$ Equation of state.

$$\frac{d}{dz}[r^2 \rho w] = 2\alpha\sqrt{\rho r^2 w^2} \tag{4}$$ Mass flux (entrainment).

$$[c r^2 w] = 1 \tag{5}$$ Contaminant flux conservation.

$$\frac{d}{dz}[r^2 \rho w^2] = r^2 (1 - \rho) \tag{6}$$ Momentum flux.

In the right-hand-side of the mass flux equation, we have chosen the Ricou-Spalding entrainment model. We were going to review other models but this one proved sufficiently good that we didn't have to. The above equations imply

$$\frac{dq}{dz} = 2\alpha\sqrt{f} \qquad \frac{df}{dz} = \frac{q}{f} \tag{7}$$

where the dimensionless momentum and mass fluxes are

$$f \equiv \rho r^2 w^2 \qquad q \equiv \rho r^2 w \tag{8}$$

and the general solution satisfies

$$f^{5/2} = \frac{5}{8\alpha}(q^2 + \beta) \tag{9}$$

where β is a constant of integration. In fact it turns out to be mathematically convenient to write everything in terms of the mass flux q rather than in terms of height z. The height is related monotonically to the mass flux by

$$z = (2\alpha)^{-4/5} \int dq \left[\frac{5}{4}(q^2 + \beta)\right]^{-1/5} \tag{10}$$

For non-zero β this function can only be written in terms of transcendental functions which are obscure enough to render the process unhelpful. However all the other interesting quantities are very simple functions of the mass flux q:

$$c = \frac{1}{q+1} \qquad \rho = \frac{q}{q+1}$$

$$w = \frac{f}{q} \qquad r = \sqrt{\frac{q(q+1)}{f}} \tag{11}$$

where f is as given above. As noted in the introduction, these simple relationships provide an excellent verification test for any computer implementation of this kind of model in the isothermal limit.

The case β=0

For β=0 z goes like $q^{3/5}$ up to a constant of integration - or, to put it in more traditional terms, q goes like $z^{5/3}$ up to a choice of origin. Furthermore f is simply a power of q. This is therefore the simplest and best known case. Turner (2) presents essentially this model, but in its analysis he looks at large z where everything goes as a power of z - effectively focusing on the β=0 solution. Fanneløp (3) goes further and derives the momentum-mass flux relationship (9) but immediately goes on to discuss point sources with β=0 and never comes back to other possibilities. Rooney and Linden (4) have discussed this model very recently, but rather than deriving the complete solution above, they assume that various fields behave as a power of z and therefore only find the solution with β=0.

Rooney and Linden note that the vertical velocity w now behaves as a power of z but that the plume radius r does not (because of the "q+1" under the square root). They therefore conclude that "non-Boussinesq effects" are present in the behaviour of the plume radius and not in the behaviour of the velocity. (Looking at our expression for the density we see that the density, ρ is effectively constant - but 1-ρ is not - for q>>1, which therefore constitutes the Boussinesq region. Therefore the "non-Boussinesqness" is encapsulated in the various factors q+1.

The case β≠0

We note that the expression for density is independent of β and so Rooney and Linden's arguments about the importance of non-Boussinesq effects generalise to all solutions. However it is worth noting that for β≠0, the more complicated relationship between f and q means that neither velocity w nor radius r are powers of z away from the Boussinesq limit.

If |β| is of order 1, then whether the β terms are non Boussinesq or not is probably a matter of semantics as the Boussinesq region q>>1 is the same as the region q>> |β|. If |β|>>1 then one can conceive of a régime $|\beta| \gg q^2 \gg 1$ the Boussinesq approximation is valid but both r and w exhibit non-power law dependencies on z - which look rather like the non-Boussinesq effects in r.

The rest of this paper concerns the case β≠0, in particular β<0. We are unaware of any previous detailed analyses of this case.

In the general case q, c, ρ, and f are all monotonic with height, but notably w and r need not be. That is to say that the plume can accelerate close to the source before slowing down, and that it may decrease in radius to a "neck" before increasing again. These properties are manifested in the case where β<0.

It is straightforward to see that in this model the point of maximum velocity is where $q=\sqrt{(-5\beta)}$ and that the neck (minimum of radius) always lies below this, at a point where q lies somewhere between this value and $1/\sqrt{3}$ times this value. (See Figure 1.)

THEORETICAL CONSIDERATIONS

Simple one-dimensional models of the kind presented above may provide an excellent representation of a fluid flow. But for this to be true, the equations of motion must be derivable from the accepted three-dimensional fluid equations, and this can only be done if an assumption of self-similarity is made. In the case in question here, the first reaction to this idea might be to wonder if the solution with $\beta<0$ can possibly be self-similar, given the complicated structure of accelerating and decelerating zones, and of increasing and decreasing radius. However this reaction is largely brought about because most self-similar analyses start with power law assumptions (cf 4 for example) and this is a much stronger assumption than that of self-similarity. In the flow in question here there is only one length scale L and one velocity scale U, and it is precisely this which admits the possibility of a self-similar solution.

Let us give a concrete illustration with the continuity equation for steady flow

$$\underline{\nabla}.\,\underline{\mathbf{U}} = 0 \tag{12}$$

(which is true for an isothermal mixture of gases in the limit where they are considered incompressible). We shall assume cylindrical symmetry with radial and vertical coordinates (r,z) and define the corresponding components of velocity (U,W). The assumption of self-similarity is that there is a plume radius function R(z) and that certain fields, when expressed in terms of the coordinates (η,z) where $\eta=r/R(z)$, are separable. That is to say a field F is self similar if it may be written $F(r,z) = F_0(z).F_p(\eta)$. The factors may be associated with a centre-line variation and a profile.

It is important to note that not all fields can be self-similar. The sum of two self-similar fields is not in general self-similar. A case in point is concentration and density: these fields can never both be self-similar. When talking of self-similar *flows* therefore, it is crucial to define which fields are required to be self-similar before you regard the flow as self-similar.

The continuity equation above may be written with no further approximation as

$$\frac{\partial}{\partial z}[\pi R^2 W] + \frac{1}{\eta}\frac{\partial}{\partial\eta}\left[\pi R\eta\left(U - \eta\frac{dR}{dz}W\right)\right] = 0 \tag{13}$$

and a self-similar form for W

$$W(\eta,z) = W_0(z)\hat{W}(\eta) \tag{14}$$

is clearly compatible with a self similar form

$$\left(U - \eta\frac{dR}{dz}W\right) = -u_E(\eta,z) = -u_{E0}(z)\hat{u}_E(\eta) \tag{15}$$

(although not necessarily with a self-similar form for the radial component U). The right hand side of this last equation is just the component of velocity orthogonal to the lines of constant η. If this self-similarity pertains, then we may immediately derive

$$\frac{d}{dz}[\pi R^2 W_0(z)] \quad = \quad 2\pi R\, u_{E0}(z) \tag{16}$$

giving a very standard looking volumetric entrainment equation. This is cast in terms of the (easily measurable) centre-line velocity, but it is also straightforward to integrate the continuity equation to get a total volume flux equation of the form

$$\frac{d}{dz}\left[\pi R^2 \int_0^\infty d\eta\; 2\eta\; W(\eta,z)\right] = 2\pi R\, u_E(z) \tag{17}$$

where

$$u_E(z) \quad = \quad u_{E0}(z) \lim_{\eta\to\infty} \left[\eta\,\hat{u}_E(\eta)\right] \tag{18}$$

(Note that the function $u_E(\eta,z)$ is odd in η and vanishes at η=0.)

We have gone through the same analysis with the momentum equation (including Reynolds stress terms) and come to the conclusion that a self similar solution for concentration and vertical velocity is possible. As in the analysis of the continuity equation, it is *not* necessary to make any power law assumption for the dependence of R on z.

We therefore conclude that there are no theoretical grounds to regard self-similarity as untenable through the neck and the point of maximum velocity. We regard this as an important stage in demonstrating the scientific soundness of the model.

An important result of this analysis is that it tells us exactly which cross-sectional averages are expected to obey the one-dimensional equations of motion - for example the area averaged velocity given above. (Various choices can be made, but most result in non-unit dimensionless constants in the 1D equations which in general arise from profile integrals. For details we refer the reader to the final report of this project. (5))

Of course the radial profile at the source is unlikely to be the similar to the asymptotic radial profile and therefore self-similarity must break down close to the source. The question is how close. This may depend on the nature of the release, but the only chance of answering this question is to appeal to experiment. To put it another way: we have shown that the self-similar model is consistent with theory, but not that it is implied by the theory. We must therefore now investigate whether the self-similar flow is consistent with experiment.

EXPERIMENTAL CONSIDERATIONS

As part of the experimental contribution to the EC project reported here (Schatzmann and Liedtke (6)) released a mixture of helium and air into calm air the wind tunnel at low initial velocity from an area source. At the source the turbulence structure is very different from that expected in a real fire. With low velocity flows great care must be taken to overcome Reynolds number problems. Here a wire mesh was placed over the source to induce turbulence. (The ideal experiment for comparison with the model would be one of a finite flux of gas from a large area source at close to zero velocity - but nevertheless fully turbulent. This is a nice trick if you can do it, but in fact, as we shall see, Schatzmann and Liedtke have come surprisingly close.)

We restrict our attention to the experiments where both vertical velocity and concentration profiles were measured. These include four releases: three from a 90mm diameter source with helium concentrations of 30%, 50% and 75% and one from a 150mm diameter source with a helium concentration of 50%. To extract centre-line values and profile radii we fitted Gaussian curves separately to each profile at each measurement height choosing the best normalisation, mean and standard deviation. This may be considered an interpolation of the data. An example (neither the best nor the worst fit) is given in Fig.2. The data are generally very well fitted by Gaussian curves except perhaps at the lowest measurement level, in this case only half a source radius above a 90mm diameter source.

The Gaussian concentration and velocity profiles allow us to construct the construct the species flux and check for conservation (Fig 3). In doing this we found that one point in particular was suspect: the highest measurement above the 150mm source seemed to be affected by material remaining in the experimental zone because of the presence of a physical ceiling on the experiment. We have tried to subtract a background level, but in the end we probably overdid this, as the species flux moved from initially being higher than that at the other levels to significantly lower - as seen in Figure 3.

These species conservation graphs give an indication of the best accuracy we may expect any subsequent analysis to achieve - typically 10 or 20% with caution over the top point on the 150mm source experiment.

The best test of the model is to extract the dimensional mass and momentum fluxes and plot $f^{5/2}$ against q^2. We did this for all data sets and found a reasonable straight line, although the best straight line in each case implied a slightly different entrainment coefficient for the 90mm and 150mm data sets. Constraining the 150mm source data to have the same entrainment coefficient as the 90mm source data resulted in the fits to f(q) shown in Figure 4. The three parameters used to fit this data are α=0.08, β(90mm)= -41.3, β(150mm)= -38.3 (Note that the theory does not require the same value of β for different data sets.)

In order now to make predictions for the original quantities, concentration, velocity, radius, etc as a function of height we need to fix one more parameter: the effective origin, which enters as a constant of integration in the expression for z(q). However it turns out that there is not an awful lot to be gained by fine tuning this parameter, and it is convenient simply to set it (for this data set) so that the model has W(z=0)=0, an idealised slow buoyant release from a large area source. The model and data now compare as shown in Figures 5-8.

CONCLUSIONS

We regard the three parameter model as an excellent fit to a much larger number of data points. The ideas of self-similarity work well for this flow, down through the neck region to within a short distance of the ground. It is therefore worth noting that the length scale L which was extracted from these experiments was in all cases much smaller than the source radius. (See Table 1). In general we would expect that the self-similarity should hold reasonably well to within a few times L of the ground.

	90mm source			**150mm source**
	30% He	**50% He**	**75% He**	**50%He**
L (mm)	7.2	8.6	9.7	13.0
U (m/s)	0.27	0.29	0.31	0.36

Table 1: The length and velocity scales extracted from the buoyancy flux in the different experiments of Schatzmann and Liedtke.

It is also important to note that without the metal gauze over the source, the experiments (6) do not produce a turbulent plume close to the source. One must therefore conclude that the structure of the turbulence close to the source, caused by the metal grid, may be quite different from the turbulence in the fully developed plume region, and that entrainment will consequently be different in this region. Nevertheless the Ricou-Spalding model seems to work well. It may be worth observing that the $\sqrt{(\rho/\rho_a)}$ factor in this model causes a suppression of entrainment close to the source where the density is lowest. (This suppression is below the already low level of entrainment expected because W is small.)

Overall we conclude that the model provides a sound scientific basis for the further development of hazard analysis models of very buoyant plumes, and that simple, one-dimensional models may prove both effective and scientifically sound in the "neck" region of a very buoyant plume. The analytic solution, in particular the linear relationship of $f^{5/2}$ and q^2, provides an excellent verification check for more complex computer models. The validation against the data of Schatzmann and Liedtke has proved effective, and that data, whilst not simulating all of the properties of fire plumes, provides an excellent check in the isothermal limit of any model which purports to describe such plumes.

Further work is needed to develop this model into one for a fire plume (the prime interest for safety studies) but development presented here illustrates the application of the EC Model Evaluation Group protocol to the initial stages of the development of such a model.

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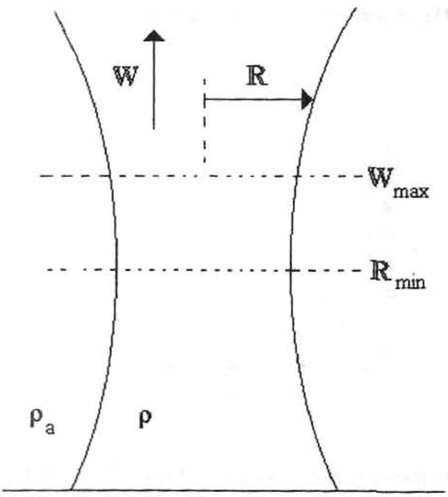
FIGURES



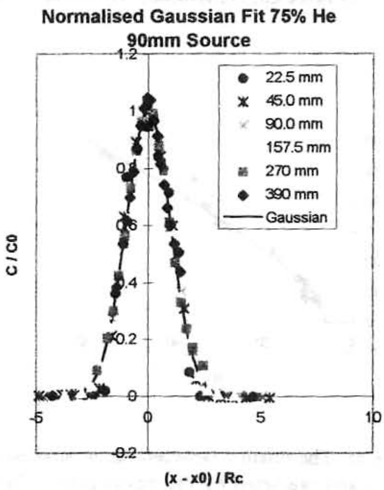


Figure 1 (left): The structure of a very buoyant plume with an area source as predicted by the model.

Figure 2 (right): An example of the concentration profiles measured at various heights above the source. The axis variables are chosen to illustrate self-similarity by collapse on to a single curve. A Gaussian curve fits the data very well.

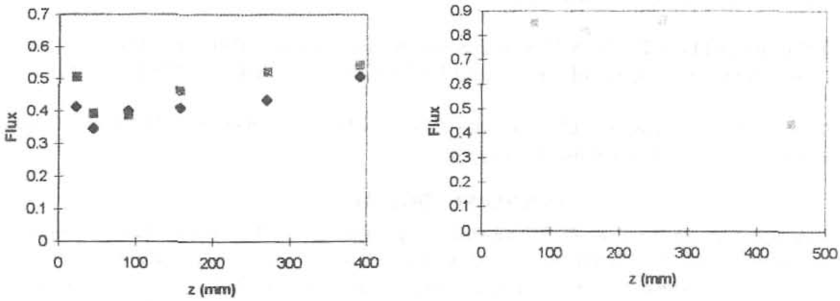


Figure 3: The total species flux (of an ethane tracer in ppm/m^2/s) as a function of height z for 90mm (left) and 150mm (right) sources. This is estimated by integrating the flux density given by interpolated Gaussian profiles. The actual species flux is constant and so these graphs give an estimate of the accuracy of our analysis technique. The highest level point on the 150mm source graph is probably erroneous, for reasons explained in the text.

[**Source concentration:** Diamond=30%He; Square=50%He; Triangle=75%He]

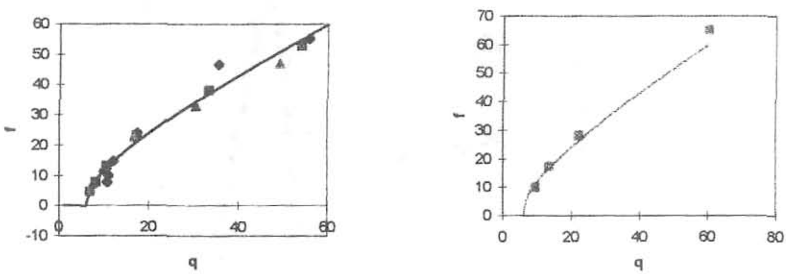


Figure 4: The relation between (dimensionless) momentum and mass flux extracted from the data (points) and predicted by the model (line). The 150mm source prediction (right) is that with the entrainment coefficient which was chosen to give the best overall fit to the 90mm source data (left).

[**Source concentration:** Diamond=30%He; Square=50%He; Triangle=75%He]

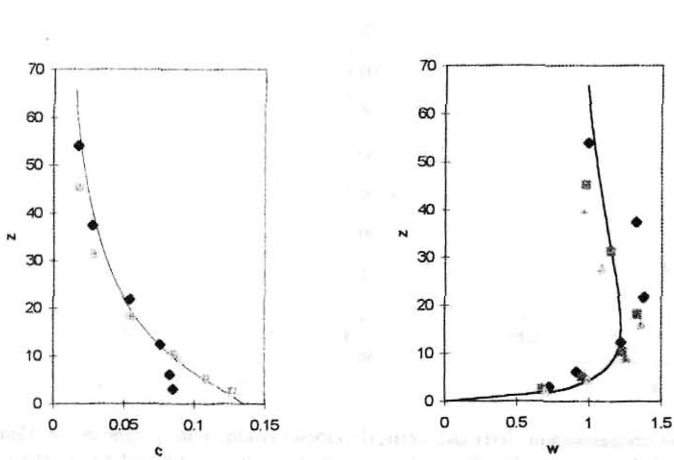


Figure 5 Variation of concentration c and velocity w with height z in the 90mm source experiments (points) and the model (line). All quantities are rendered dimensionless.

[**Source concentration:** Diamond=30%He; Square=50%He; Triangle=75%He]

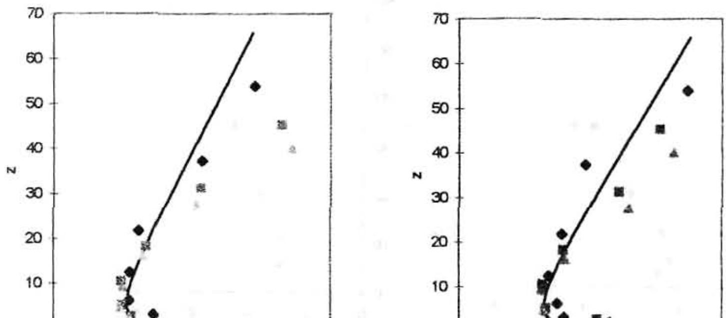


Figure 6: The plume radius (Gaussian width of concentration and velocity profiles) in the 90mm source data (points) and the model (line).

[**Source concentration:** Diamond=30%He; Square=50%He; Triangle=75%He]

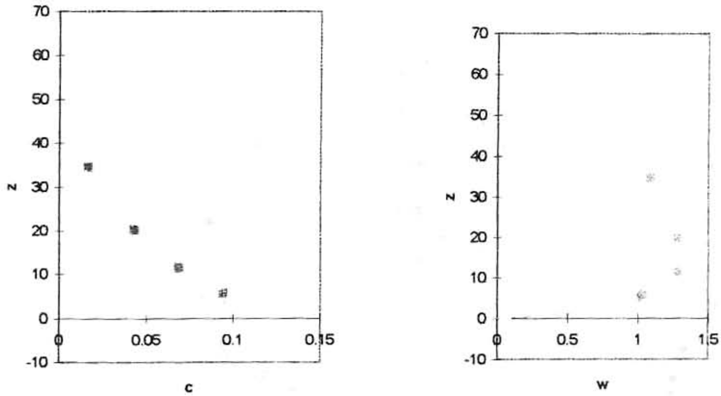


Figure 7 Variation of concentration (left) and vertical velocity (right) with height in the 150mm source experiments and the model. (The fit employs parameter values optimised from fitting the 90mm source data.)

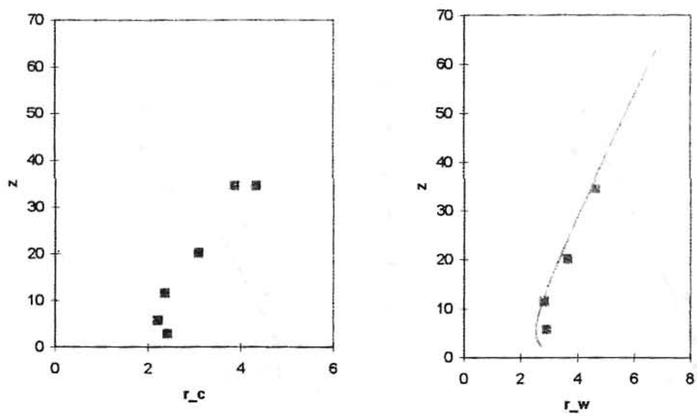


Figure 8: The plume radius (Gaussian width of concentration and velocity profiles) in the 150mm source data and the model. . (The fit employs parameter values optimised from fitting the 90mm source data.)