DEPOSITION FROM PARTICULATE CLOUDS

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Modifications to Sutton's dispersion equations are used to estimate the rate of deposition of particles. For particles larger than 10 micron radius a gravitational factor is used. For smaller particles a notional deposition velocity derived from experimental data is used.

Stokes Law

The trajectory of a particle in a cloud is determined by its displacement under gravitational forces and the effects of aerodynamic forces.

For solid or liquid particles the density is so much greater than that of air that the latter may be neglected and the well known Stokes Law becomes

$$V = 2r^2 g \rho / 9\mu \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

The terminal velocity, V, is reached very soon after the particle begins to fall so that V may be taken as the constant rate of fall.

It is necessary to take note of the limitations of Stokes Law. It applies to smooth, small spheres of such size and density that the Reynolds number is about unity or less falling through still air. This arises because the drag coefficient is dependant on the Reynolds number which for a sphere may be defined as

$$R = 2Vr/\eta \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

From equation (1), for a particle of density 2.5 g/cm^3 (a common value of many solids), with g = 981, and $\mu = 1.8 \times 10^{-4}$ poises for air

$$V = 0.03\, r^2 \qquad \text{with } r \text{ in microns} \quad . \quad . \quad . \quad . \quad . \quad (3)$$

By substitution in (2) and putting R = 1 it is found that Stokes Law applies to smooth spheres of density 2.5 of a radius of 1.3 microns or less, which have a terminal velocity of 0.07 cm/sec or less.

Rate of fall of large particles

With larger spheres, more than a few microns radius, the drag coefficient increases as air flow around the sphere ceases to be laminar. In this, the aerodynamic region, McDonald (1,2) has devised a method of computing terminal velocities taking into account the various factors, and has given terminal velocities for spheres of density 2.5. Similarly Hage (3) has given them for spheres of density 5.0.

The values given by McDonald and Hage do not agree exactly. There is also an apparent discrepancy between the two. Hage's values for the heavier particle give a lower terminal velocity than McDonald's. But there is not a very great difference in velocities for particles of different densities.

Since most solid particles have a density between 1 and 5, and because of the other uncertainties involved it is suggested that no great error (less than a factor of 1.5) would be incurred by using the logarithmic equation

$$\log V = 1{\cdot}224 \log r - 0{\cdot}536 \quad . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; (4)$$

for particles in the size range 10-1000 microns radius.

If equation (4) is compared with Stokes Law, equation (3), it will be seen that the former gives a terminal velocity about 20-25 times smaller than the latter.

Particles larger than 1000 microns radius will fall at a terminal velocity greater than 10 m/sec and therefore would reach the ground near the source, even in a high wind, unless they were released from a great height. This, of course, would not apply to particles of an abnormally low apparent density such as might be produced by sublimation.

These results apply specifically to smooth spheres. The effect of a departure from spherical is, on average, to reduce the terminal velocity by a factor of about $\frac{2}{3}$.

Deposition from a continuous point source

For a continuous elevated source containing particles in this range of sizes, Sutton's equation may be modified by introducing a term to give a downward tilt to the centre line of the plume corresponding to the gravitational fall of the particles. Such an equation for concentration at ground level is

$$\chi_{(x,y,0)} = \frac{2Q}{\pi C^2 u x^{(2-n)}} \exp\left[-\frac{x^{(n-2)}}{C^2}\left\{y^2 + (h - xV/u)^2\right\}\right] \quad . \; . \; . \; . \; (5)$$

If it is assumed that each particle when it reaches the ground is retained there, the rate of deposition on the ground is given by

$$D' = V\chi_{(x,y,0)} \quad . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; (6)$$

This clearly overestimates the deposition rate because the cloud is continuously being depleted by the deposition. In order to take this into account Csanady (4) proposed an approximate depletion factor defined as

$$F_{(x,y,0)} = 1 - 1/\left\{(1-n/2)(hu/xV - 1) + 2\right\} \quad . \; . \; . \; . \; . \; . \; (7)$$

So that the deposition rate becomes

$$D = FV\chi_{(x,y,0)} \quad . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; (8)$$

The assumption that each particle is retained on reaching the ground may well be a reasonable approximation if the surface is of a retentive nature, such as a wet or "absorptive" surface like grass. The production of "sand devils" by a wind sweeping over a desert or the drift of sand particles across a smooth surface demonstrate that the assumption may not always be valid. In such cases the deposition rate must be modified by an adherence factor which, it seems, can only be determined experimentally.

Deposition from an instantaneous point source

A similar treatment may be applied to an instantaneous point source using Sutton's equation for an elevated source modified for the gravitational effect in place of equation (5). In that case the quantity deposited is derived from the integral of the ground concentration.

Deposition of small particles

For particles less than about 10 micron radius, or those falling at less than 1 cm/sec the gravitational effect becomes insignificant compared with movements due to air turbulence.

Other complications arise, particularly in respect of the extent to which particles come into contact with the ground and are there retained. No satisfactory explanation of the mechanisms involved is available. It is necessary therefore to rely on the limited amount of experimental data which has been published.

Gregory (5) studied the experimental data obtained by previous workers on the numbers of seeds, spores and pollen deposited.

He considered that the Sutton equations were adequate descriptions of the concentrations of particles. The problem then was, given a known ground level concentration, what number of spores etc. were deposited, since, in his view, it was more appropriate to regard the cloud as a suspension rather than as a shower of particles falling under gravity.

He concluded that the deposition occurred through the falling of the particles contained in a thin boundary layer in which there was no turbulence, a layer originally suggested by Brunt (6). This meant that all the particles in a static layer of thickness d would be deposited.

From the experimental data he determined that an average value was d = 0.05 cm. It should be pointed out that this does not mean that this is the actual thickness of the stagnant layer. It is an empirical quantity which expressed the total effect of all the factors affecting deposition.

This value of d may then be used with the Sutton equations to give the quantity deposited, that is the quantity in a horizontal ground level plane of thickness 0.05 cm.

Chamberlain (7) studied the fall out of radio-active particles. He defined a velocity of deposition, V_g, as the quotient of total deposition divided by the volumetric concentration in the cloud. Thus it corresponds to the terminal velocity in the previous equations.

He computed values of V_g from experimental data obtained by Bullas (8), Stewart (9), Booker (10) and Megaw and Chadwick (11). Results were very variable giving values for V_g from 0.01 to 0.2 cm/sec. Perhaps the most interesting are the results by Megaw and Chadwick using a cloud of uranium oxide with particle sizes 0.2 micron or less. In outdoor experiments for collection on filter paper $V_g = 0.04 - 0.07$; for collection on grass $V_g = 0.20$ cm/sec.

In another paper, Chamberlain (12) proposed a depletion factor

$$Q_d = Q \exp\left[-4V_g x^{n/2} / n u \pi^{1/2} c\right] \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (9)$$

which can be inserted in the Sutton equations in place of Q. The rate of deposition is then given as before by multipl $\chi_{(x,y,0)}$ by the deposition velocity V_g.

For practical purposes it is suggested that for deposition outdoors on grass the value $V_g = 0.2$ cm/sec should be tried. However it should be borne in mind that there are still great uncertainties and lack of experimental confirmation. In particular the possible effects of surfaces other than grass and of buildings, elevated vegetation etc. are completely unknown.

SYMBOLS USED

V = Terminal velocity (LT^{-1})

r = radius of particle (L)

g = gravitational acceleration (LT^{-2})

ρ = density of particle (ML^{-3})

μ = dynamic viscosity of air ($ML^{-1}T^{-1}$)

R = Reynolds number (dimensionless)

η = kinematic viscosity (L^2T^{-1})

$\chi_{(x,y,0)}$ = concentration of particles (ML^{-3}) at point $(x,y,0)$

x = distance from origin, along mean wind direction (L)

y = cross wind, horizontal distance (L)

z = vertical distance from ground (L)

Q = source strength (ML^{-3})

u = mean wind velocity (LT^{-1})

C = Sutton's diffusion coefficient (L)

n = Sutton's stability parameter (dimensionless)

h = height of source (L)

D' = undepleted deposition rate ($ML^{-2}T^{-1}$)

F = depletion factor, equation (7) (dimensionless)

D = depleted deposition rate ($ML^{-2}T^{-1}$)

d = thickness of deposition layer (Gregory) (L)

V_g = deposition velocity (Chamberlain) (LT^{-1})

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