

# ESTIMATION OF THE EXTENT OF HAZARD AREAS ROUND A VENT

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## SUMMARY

This paper reviews and unifies some of the more simple published work relevant to predicting the spread of toxic and flammable gases or vapours from a vent. After discussing the measure of concentration appropriate to each hazard, it considers continuous point sources of emission discharging into quiescent and turbulent surroundings. The practical application of the results is discussed and exemplified.

### Introduction

Calculation of the spread of effluent from a vent is frequently hindered by the difficulty of finding published information, and by its complexity when found. The present paper reviews some of the more simple results available in the literature in the hope that these may be useful to those practically concerned with the problem of siting and erecting vents to discharge toxic or flammable substances to the atmosphere.

The treatment will be restricted to continuous point sources of emission, discharging into both quiescent and turbulent atmospheres. Separate consideration will be given to the effects of momentum and buoyancy.

### Comparison of Toxic and Flammability Hazard

At the outset it will be apparent that while atmospheric dilution of toxic and flammable materials is similar in principle, there are differences of practical importance, *e.g.* concentration, and range. Because quite low concentrations of a toxic effluent may be harmful, the toxic hazard is a long-range problem. It is generally concerned with the effects of low concentrations averaged over fairly long periods of time. By contrast, since the lower limit of flammability occurs at a relatively high concentration, the flammability hazard is essentially short-range. In this case to avoid the risk of inflammation it is necessary to keep the maximum instantaneous concentration below the lower limit.

The distinction between the two problems with respect to time scale of concentration measurement is particularly important. For man, working moderately hard, it takes about two minutes for the air in the lungs to equilibrate with the atmosphere, and usually a matter of several hours for the attainment of complete equilibrium with the whole body, so that instantaneous concentrations of toxic substances are not likely to be of great interest. On the other hand, ignition of flammable materials proceeds rapidly, and eddies of short duration but high concentration may ignite even though the time-average concentration is well below the limit of flammability. Unfortunately this at once introduces uncertainty, because the form of the instantaneous concentration frequency distribution curve is generally not known. However, at high concentrations in the centre of a well-defined plume, the distribution about the time-mean is probably fairly symmetrical, so that instantaneous concentration is unlikely to exceed twice the time-mean.

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### Concentration Fluctuations

Estimates of the relation between time-mean and maximum instantaneous concentrations seem to be sparse in the literature.

Katan<sup>1</sup> when considering the flammability hazard in fuelling aircraft assumed the maximum instantaneous concentration at any point to be 2.2 times the maximum 3 min time-mean concentration at the same point. No evidence was given to support the choice of the factor of 2.2, other than that which may be inferred from a series of explosimeter readings for one typical experiment.

Evidently from the magnitude of Katan's factor it must be based on near-symmetrical fluctuation distributions. However, casual observation of steam jets or chimney plumes would suggest that this situation does not prevail at the edges, particularly under "looping" conditions. This view is strengthened by field measurements made by Gosline<sup>2</sup> of low concentrations of nitrogen oxides at ground level five and ten stack heights downwind of an elevated source. Using a photo-electric analyser with a sampling time of 10 s he found no response during 75% of the time of observation. Thus the distribution curve under these conditions was clearly unsymmetrical, and the author concluded that the instantaneous maximum could have been 50 times the time-mean.

The disturbing difference between Katan's estimate and the experiment reported above serves very well to indicate the caution necessary in applying results from one situation to another in this field. Obviously errors of this magnitude may make involved calculation of instantaneous concentrations practically worthless. This is particularly true in toxicity problems involving low concentrations at the edge of a plume. However, as previously pointed out, the instantaneous concentration is rarely required in this application. Furthermore wide variations in concentration at a fixed point are more likely to occur when atmospheric conditions cause rapid dilution of a plume; a situation which may be of little interest in fixing hazard areas, if these must apply equally to conditions of poor dilution.

The relation between average values of time-mean concentrations measured with different sampling times has been considered by Wippermann<sup>3</sup> for the special case of the maximum ground level concentration from an elevated source. This treatment is based in part on observed fluctuations in wind speed and direction which could also be applied to concentration measurements other than the maximum at ground level. The final result is a formula for the ratio of the average time-mean concentration for a given sampling time to the time-mean concentration for infinite sampling

time. Values of this ratio were tabulated for periods ranging from 2 min to four days. These agree reasonably with experimental concentration measurements by Stewart, Gale, and Crooks,<sup>4</sup> and predictions by Meade.<sup>5</sup> The results are of use in toxicity problems but cannot be reliably extrapolated to the short sampling time involved in flammability hazards. In fact Wippermann's formula predicts infinite average instantaneous concentrations, *i.e.* perpetual hazard.

Concentration fluctuations occurring in turbulent jets under laboratory conditions have been estimated by Hawthorne, Weddell, and Hottel<sup>6</sup> for turbulent diffusion flames, and measured by Rosenweig, Hottel, and Williams<sup>7</sup> for a jet of smoke by using a light scattering technique. A related property investigated by Kristmanson and Danckwerts<sup>8</sup> is the maximum penetration of a given concentration of jet fluid into its surroundings. This was estimated from photographs of an alkaline jet discharging into a tank of acid in which the stoichiometric surface was visualised by an indicator.

**Discharge into a Quiescent Atmosphere**

*The turbulent momentum jet*

The most studied system is the turbulent jet with negligible buoyancy, compared with its inertia, issuing from an orifice into stagnant surroundings. Under these conditions the following experimental observations have been made:

- (a) The jet is conical, apparently diverging from an equivalent point source upstream of the orifice,
- (b) dilution is by turbulent mixing,
- (c) time-mean velocity and concentration profiles are similar after 10 diameters, and approximately Gaussian in form,
- (d) the concentration profile is wider than the velocity profile,
- (e) the radial spread of mass and velocity increases with decreasing density of jet fluid,
- (f) entrainment decreases with increasing Reynolds number up to 20 000, and thereafter remains constant.

As it is diluted, the issuing jet shares its momentum with entrained gas so that momentum is conserved within the jet and the momentum flux at any plane normal to the axis is constant. By equating the momentum flux at a plane axial distance  $x^*$  from the equivalent point source to that at the orifice, it follows that the ratio of mass flow at the two levels is given by:

$$\frac{m_x}{m_o} = k_1 \frac{x}{d_o} \left( \frac{\bar{\rho}_x}{\rho_o} \right)^{\frac{1}{2}} \quad (1)$$

where  $k_1$  is a constant of proportionality depending on the jet geometry, and the forms of the velocity profiles at  $x$  and the orifice.

The constant  $k_1$  in equation (1) has recently been measured by Ricou and Spalding<sup>9</sup> for jets of hydrogen, air, carbon dioxide, and propane discharging into air from a shaped orifice. These investigators obtained a value of 0.32 and established the validity of equation (1) up to 418 diameters.

Since for constant rate of emission, the average volumetric concentration at any level is inversely proportional to the volumetric flow at the same level, it follows that the ratio of the average time-mean concentration at  $(c_x)\bar{x}$  to the initial value ( $c_o$ ) is given by:

$$\frac{\bar{c}_x}{c_o} = \frac{1}{k_1} \frac{d_o}{x} \left( \frac{\bar{\rho}_x}{\rho_o} \right)^{\frac{1}{2}} \quad (2)$$

Likewise, if a Gaussian distribution of concentration is assumed, the time-mean concentration at a point axial distance  $x$ , and radial distance  $r$  from the apparent point source is given by:

$$\frac{\bar{c}_{xr}}{c_o} = k_2 \frac{d_o}{x} \left( \frac{\bar{\rho}_x}{\rho_o} \right)^{\frac{1}{2}} \exp - \left( \frac{k_3 r}{x} \right)^2 \quad (3)$$

Values of  $k_2$  and  $k_3$  calculated from experiments on mass or heat transfer reported in the literature are given in Table I.

TABLE I.—Values of the Constants  $k_2$  and  $k_3$  in Equation (3)

Investigators	Jet fluid	Entrained fluid	$R_o$ at orifice	$k_2$	$k_3$
Keagy and Weller <sup>10</sup>	CO <sub>2</sub>	air	50 000	5.4	9.2
	N <sub>2</sub>	air	27 000	5.4	7.9
	He	air	3 400	4.1	5.3
Hinze and van der Hegge Zijnen <sup>11</sup>	air + 1.1% town gas	air	67 000	5.3	8.8
	hot air (30°C above ambient)		67 000	5.3	8.8
Sunavala, Hulse, and Thring <sup>12</sup>	air + nitrous oxide tracer	air	29 000–57 000	4.5	7.1
	hot air (320°C above ambient)	air	18 000–25 000	4.5	7.1
	water + 1% NaCl	water	—	5.2	7.9
Forstall and Gaylord <sup>13</sup>	water + dye	water	12 000	4.8	7.4
Kristmanson and Danckwerts <sup>8</sup>	hot air (15°C above ambient)	air	30 000–60 000	5.3	7.8
	hot air (300°C above ambient)	air	10 000–20 000	5.0	6.4
	hot air (17°C above ambient)	air	—	5.9	7.7
Corrsin and Uberoi <sup>14</sup>	hot air	air	—	4.8	7.1
Squire <sup>15</sup>	hot air	air	—	5.9	7.7
Ruden <sup>16</sup>	hot air	air	—	—	—
Hawthorne, Weddell, and Hottel <sup>6</sup>	air	air?	3000–8000	4.4	—

Fig. 1 shows a plot of  $k_2$  against Reynolds number at the opening for the cases in which the latter was given or could be calculated. For experiments involving a range of Reynolds numbers the mean was taken. It will be seen that, apart from the work of Sunavala *et al.*<sup>12</sup> (marked with crosses), the results correlate well showing decreasing entrainment with increasing Reynolds number until constancy is achieved above a Reynolds number of about 20 000. The broken line is the mean of two experimental curves obtained by Spalding for  $1/k_1$  measured at 14 and 26 diameters respectively. This shows the same dependence on Reynolds number.

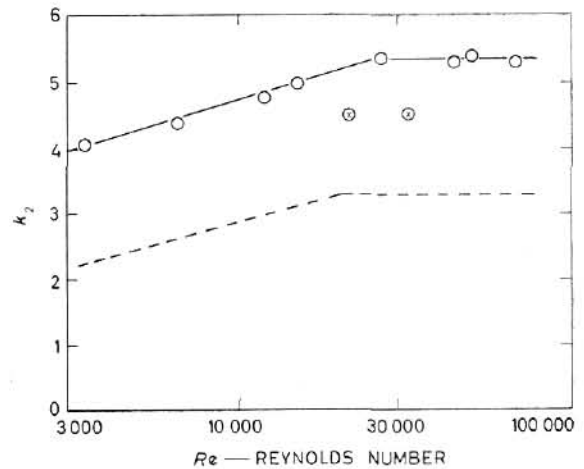


Fig. 1.—Graph of  $k_2$  against Reynolds number at the orifice

\* Symbols have the meaning given them on p. 13.

The anomalous results of Sunavala *et al.* raise a relevant practical consideration. They probably arise from non-uniformity of the velocity profile at the opening. Most investigators have used shaped nozzles or orifices to get a uniform velocity at the opening, whereas Sunavala used either a long tube or stepped tubes made by inserting a short length of narrow bore tubing in the end of a long tube. The existence of a non-uniform velocity profile at the opening causes the initial momentum flux to exceed the product of mass flow and average velocity, thereby giving a lower value of  $k_2$ . For the easily calculated case of a parabolic distribution of velocity,  $k_2$  is reduced by about 14%. This is also the amount by which Sunavala's results lie below the mean line. The point of practical importance is that non-uniformity of initial profile causes more rapid dilution for a given average velocity of efflux.

Fig. 2 shows how the observed spread of a jet correlates with the density ratio of jet fluid to ambient atmosphere. However this could be spurious since low density fluids have always been used at low Reynolds number. Based on the results in Table I, a correlation also exists between  $k_3$  and Reynolds number.

It will be seen from Table I that the maximum value of  $k_2$  is 5.9, while the minimum value of  $k_3$  is 5.3. These values correspond to the maximum axial and radial spreads respectively. Thus a conservative estimate of the time-mean concentration at a point of  $xr$  results from the substitution in equation (3) of  $k_2 = 6$ , and  $k_3 = 5$ .

Less information is available regarding the maximum instantaneous concentration at a point. The results of the neutralisation experiments of Kristmanson and Danckwerts<sup>8</sup> may be summarised as:

$$\frac{c_{xr}}{c_0} \leq 7.0 \frac{d_0}{x} \exp - \left( \frac{2r}{x} \right)^2.$$

Comparison with the results of the mixing experiment by the same investigators, given in Table I, indicates an extension of the axial range of hazard by 50%. Therefore to estimate maximum instantaneous concentrations, it is suggested that equation (3) is used with  $k_2 = 9$ , and  $k_3 = 2$ .

Further support for the above value of  $k_2$  comes from the measurements of fluctuations in temperature and concentration reported by Corrsin and Uberoi<sup>14</sup> and Rosenweig *et al.*<sup>7</sup> respectively. The former measured the root-mean-square temperature fluctuations about the time-mean at various positions in a jet of hot air. These results should be similar to concentration on fluctuations in gaseous jets. The r.m.s. temperature fluctuation on the axis was found to be about 16% of the time-mean value at the same point. Thus, if the

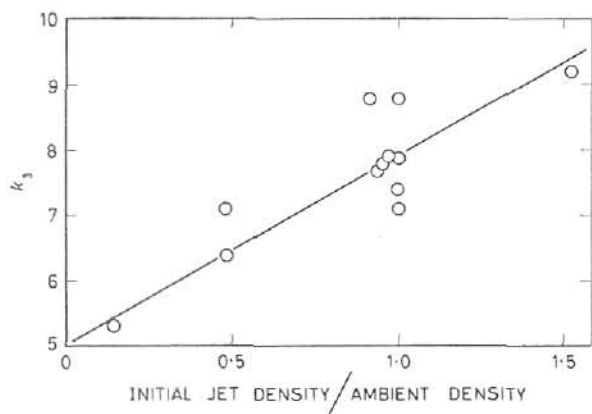


Fig. 2.—Correlation of density ratio of the jet fluid with ambient temperature

time-mean concentration is given by equation (3) with  $k_2 = 6$ , and the distribution of fluctuations is Gaussian (as is probable for the axis), then the probability of the instantaneous concentration exceeding that predicted by  $k_2 = 9$  is 0.1%. Rosenweig measured the concentration fluctuations in a smoke jet up to 40 diameters from the source. These fluctuations will be higher than for gaseous jets because of the relatively low (Brownian) diffusivity of the particles employed. It was found that on the axis the r.m.s. fluctuation was about 20% of the time-mean concentration. Thus with the same assumptions as before, the probability of the instantaneous concentration exceeding that predicted by  $k_2 = 9$  is 0.6%. Because of the uncertain time distribution of fluctuations at the edge of the jet, no conclusions will be drawn regarding the suitability of the proposed value of  $k_3$ .

#### The buoyant plume

In this case gas emerges into surroundings of different density, and motion is predominately due to buoyancy.

Experimentally it is found that:

(a) Under turbulent conditions the plume is conical, apparently diverging from an equivalent point source.

(b) The position of the point source depends on the point of transition from laminar to turbulent motion.

(c) The time-mean velocity and concentration profiles are similar and approximately Gaussian in form.

Under adiabatic conditions the issuing plume shares its buoyancy with entrained gas, hence buoyancy is conserved within the plume, and the buoyancy flux remains constant with distance. It follows therefore that momentum flux increases with distance.

The critical distance above which turbulence sets in is a function of viscosity,  $\mu$ , and density,  $\rho_a$ , of the ambient atmosphere, volumetric flow-rate,  $Q$ , of the effluent, the difference in density,  $\Delta\rho_0$ , between the effluent and the ambient atmosphere, and the acceleration because of gravity,  $g$ . From the experiments of Yih<sup>17</sup> on burning cigarettes, this distance is:

$$\left( \frac{\mu^3}{\rho_a^2 Q \Delta\rho_0 g} \right)^{\frac{1}{3}} \times 10^5.$$

In most practical situations it is quite small so attention will be confined to turbulent dilution.

Assuming similarity of profiles, and a Gaussian distribution of concentration, it follows from conservation of buoyancy, and application of Newton's second law of motion, that the time-mean concentration at a point axial distance  $x$  greater than 5 diameters and radial distance  $r$  is given by:

$$\frac{c_{xr}}{c_0} = k_4 \left( \frac{Q^2 \rho_a}{\Delta\rho_0 g x^5} \right)^{1/3} \exp - \left( \frac{k_5 r}{x} \right)^2 \quad (4)$$

where  $\rho_a$  is the ambient density.

Direct experimental proof of equation (4) seems to be lacking in the literature, but values of  $k_4$  and  $k_5$  may be obtained from heat transfer studies. The most reliable of these were carried out by Yih<sup>17</sup> who measured the temperature distribution above small premixed gas flames and found  $k_4 = 11$ , and  $k_5 = 8.4$ . This value of  $k_4$  is supported by the work of Hird, Bigmore, and Pickard,<sup>18</sup> who measured temperatures under ceilings above diffusion flames of methylated spirit burning from trays of 6 in. to 3 ft dia. The axial temperature was expressed as an implicit function of distance and convective heat output. From this equation, after making suitable allowance for radiation heat losses from the flame, it has been possible to derive a value of  $k_4 = 11.6$ . However, this value is necessarily less accurate than that obtained by Yih.

No work is known on the relation between time-mean and maximum instantaneous concentrations in buoyant plumes. Comparison with the momentum jet suggests it is probably adequate to use  $k_4 = 17$  in the calculation of maximum instantaneous axial concentrations.

#### Combined effects of momentum and buoyancy

A momentum jet of density different from its surroundings ultimately degenerates into a buoyant plume. The transition occurs gradually and it is possible to distinguish three regimes: the turbulent jet, the transition region, and the fully developed buoyant plume. The extent of each region has to be determined experimentally.

It is convenient for calculation purposes to assume the transition occurs sharply at a point, and then to smooth the resulting concentration profile. The position of the transition point will depend on the ratio of inertia forces to buoyancy forces, which may be regarded as a modified Froude Number ( $F$ ) given by the expression:—

$$F = \frac{\rho_0^{3/2} v_0^2}{\rho_a^{1/2} \Delta \rho_0 g d_0}$$

where  $v_0$  is the initial velocity of the jet.

An approximate estimate of the transition point may be obtained by solution of equations (3) and (4) for the same concentration assuming a negligible difference in  $x$ . Thus, on substituting  $k_2 = 5.4$  and  $k_4 = 11$ , the transition point occurs at  $2.4 F^{1/2}$  diameters from the momentum source.

The above value is confirmed by the experimental work of Ricou and Spandling<sup>9</sup> who measured entrainment by vertical jets and flames. The point of equal entrainment on extrapolated lines through experimental points in the jet and plume regimes occurs at  $2.3 F^{1/2}$  diameters downstream. The momentum jet persists at least to a distance of  $0.5 F^{1/2}$  diameters and the buoyant plume is fully developed beyond  $3 F^{1/2}$  diameters.

Since dilution is more rapid with distance in a buoyant plume than in a jet it follows that the jet formula gives a conservative estimate of concentration provided buoyancy forces do not oppose the momentum forces.

#### Discharge into a Turbulent Atmosphere

Dilution proceeds more rapidly in a turbulent atmosphere because of the mixing action of eddies already present. These eddies are characterised by fluctuations of windspeed about the time-mean value both in magnitude and direction. They may be enhanced or damped by buoyancy forces arising from rapid changes in position.

If a packet of air changes height suddenly, its temperature will alter adiabatically according to the change in pressure which it experiences. If this temperature is different from that of its surroundings, buoyancy forces are generated which may either assist or oppose motion. These forces are only absent if the temperature gradient of the atmosphere is adiabatic, *i.e.*  $-0.01^\circ\text{C}/\text{m}$ , the so-called neutral atmosphere. Under inversion conditions the gradient is positive, eddies are damped by buoyancy forces and mixing is reduced; whereas in super-adiabatic or strong lapse conditions, eddies are enhanced and mixing proceeds rapidly.

#### Discharge with negligible buoyancy and momentum

Experimentally it is observed that:

(a) Plumes remote from the ground have an elliptical cross section, and

(b) the concentration profiles are approximately Gaussian.

According to Sutton<sup>19</sup>, the standard deviations,  $\sigma$ , of the time-mean concentration profiles in the two principal directions are given by:

$$\sigma_y^2 = \frac{1}{2} C_y^2 x^{2-n}$$

$$\sigma_z^2 = \frac{1}{2} C_z^2 x^{2-n}$$

where:  $C$  is a generalised diffusion coefficient

$x$  is the distance downwind from the source

$n$  is a parameter theoretically having values between 0 and 1.

The parameter  $n$  is related to the transport properties of the atmosphere and may be derived from the velocity profile, or roughly estimated from meteorological conditions and published data (see Table II and Refs 19, 20, 21).

TABLE II.—Values of  $n$  and  $C$  for Various Meteorological Conditions above Downland

Based on values given by Sutton<sup>20, 21</sup>

Average values for the first ten metres of atmosphere above ground

Meteorological Condition	$n$	$\bar{C}_y(0-10)$			$\bar{C}_z(0-10)$		
		( $\text{m}^{n/2}$ )	( $\text{ft}^{n/2}$ )	( $\text{cm}^{n/2}$ )	( $\text{m}^{n/2}$ )	( $\text{ft}^{n/2}$ )	( $\text{cm}^{n/2}$ )
Large lapse rate	1/5	0.37	0.42	0.59	0.21	0.24	0.33
Neutral conditions	1/4	0.21	0.24	0.37	0.12	0.14	0.21
Moderate inversion	1/3	0.13	0.16	0.28	0.08	0.10	0.17
Large inversion	1/2	0.11	0.15	0.35	0.06	0.08	0.19

Values for heights greater than 10 metres

Neutral conditions only;  $n = 1/4$

Height (m) Ratio  $C_y(h)/\bar{C}_y(0-10)$

20	0.75
40	0.66
60	0.61
80	0.57
100	0.54
150	0.49
200	0.46
300	0.40

Above 25 metres  $C_x = C_y$  (isotropic turbulence)

For certain special conditions  $C$  may be calculated from the statistical theory of turbulence. The values reported by Sutton were based mainly on experiments performed over a few hundred metres of downland under neutral conditions, with suitable theoretical extrapolation to include other situations. Increasing the surface roughness increases  $C$ ; so values appropriate to smooth surfaces should give conservative estimates of concentration acceptable for assessing hazards. Separate coefficients are defined for each of the principal directions because diffusion may be asymmetric.

More recent work reported by Pasquill<sup>28, 29</sup> gives alternative methods for estimating  $\sigma$  from meteorological data. The results may be applied in the same way as the Sutton values.

From the mass balance involving the rate of emission,  $Q$ , the mean wind speed,  $\bar{u}$ , and the assumed form of the concentration profiles, the concentration can be calculated at any point remote from the ground. After allowing for perfect reflection by the ground, the concentration at point  $x, y, z$ , arising from a source at the origin of the co-ordinate system and a height  $h$  above ground, is given by:

$$\bar{c}(x, y, z) = \frac{Q \exp - (y^2/C_y^2 x^{2-n})}{\pi C_y C_z \bar{u} x^{2-n}} \times \left[ \exp - \frac{(z)^2}{C_z^2 x^{2-n}} + \exp - \frac{(z+2h)^2}{C_z^2 x^{2-n}} \right] \quad (5)$$

From equation (5) it follows that:

(a) Maximum ground level concentration:

$$\bar{c}_{\text{mgl}} = 0.234 \frac{Q}{\bar{u}h^2} \left( \frac{C_z}{C_y} \right) \quad (6)$$

(b) Distance from source at which maximum ground level concentration occurs:

$$x_m = \left[ \frac{h^2}{C_z^2} \right]^{1/(2-n)} \quad (7)$$

(c) The maximum concentration at a distance  $x$  from the source (*i.e.* centre line concentration) is:

$$\bar{c}(x_{00}) = \frac{Q}{\pi C_y C_z \bar{u} x^{2-n}} \left[ 1 + \exp\left( \frac{-4h^2}{C_z^2 x^{2-n}} \right) \right] \quad (8)$$

The above concentrations are time-mean values and correspond to a sampling time of three minutes if Sutton's values of  $C$  are used.

TABLE III.—Ratio of Average Maximum Ground Level Concentration for a Sampling Time  $t$  to that for a Sampling Time of three minutes (after Wippermann<sup>3</sup>)

$t$ (min)	Ratio
3	1
10	0.77
20	0.70
30	0.67
60	0.63
360	0.47

The maximum ground level concentration for longer sampling times may be estimated from Table III which is based on the time variation of the group  $\frac{1}{\bar{u}} \left( \frac{C_z}{C_y} \right)$  proposed by Wipperman<sup>3</sup>. From the work of Stewart *et al.*<sup>4</sup> the axial concentration is inversely proportional to the one-fifth power of sampling time for times ranging from two to 13 minutes.

Instantaneous concentrations may be higher than the time-mean and on the plume axis are unlikely to exceed twice the three minute time-mean.

Experimental surveys which may be used to test the above formulae have been carried out by Katan<sup>1</sup>, Gosline<sup>2</sup>, and Stewart *et al.*<sup>4</sup> Katan measured concentrations of petrol close to vents in aircraft fuel tanks, during refuelling under a variety of meteorological conditions ranging from unstable to moderate inversion. The concentration was measured by explosimeter and was essentially instantaneous. He limited his comparison with Sutton to showing a similar form of decay for maximum concentration with distance. However, from his experimental results it can be shown that the maximum recorded concentration at sampling points on the plume axis was less than the three minute mean predicted by equation (8) for neutral conditions, assuming the effluent gases to be saturated with petrol vapour.

Gosline measured ground level concentrations of oxides of nitrogen downwind of a stack 80 ft high. His observations were restricted to stations placed five and ten stack heights from the source under mainly neutral or unstable conditions. He concluded that the measured concentration agreed with Sutton's predictions on average. However, from his calculations this appears to be more the result of judicious averaging of compensating errors than the correctness of the original formula. For instance at five stack heights under neutral conditions in moderate wind the calculated result was less than one hundredth of that actually found. This was an

exceptional discrepancy but it serves to show that the formulae are unreliable at high values of  $z/x$ . Generally estimates were too high under unstable conditions and too low under neutral conditions.

Stewart *et al.* surveyed the radioactive plume from the Harwell reactor BEPO. They compared their findings with Sutton's equations by calculating diffusion coefficients from observed concentration profiles. By reconvertng their results to concentrations, it may be shown that by the use of equation (8) with the values in Table II over-estimates the concentration for neutral and stable conditions. For unstable conditions the concentration found was about twice the predicted value. One notable feature of this work was the high values found for  $C_z$ , which were attributed to fluctuations in plume caused by the gustiness of the wind. The maximum ground level concentration occurred at about 17 stack-heights down wind.

Summarising the evidence, it appears that:

(i) Equation (8) gives conservative estimates of maximum concentration at any distance from the source under neutral and moderately stable conditions.\*

(ii) Equation (5) is unreliable for high values of the ratio  $z/x$ : under neutral and stable conditions, it should not be used for  $z/x$  greater than  $\frac{1}{10}$ .

(iii) For situations other than those covered by (i) and (ii) above, the predicted concentration is not likely to differ from the actual concentration by more than a factor of three.

#### Discharge with appreciable buoyancy and momentum

In this section it will be convenient to consider a discharge which rises above the source; for a falling effluent the rise may be considered negative.

Observation of smoke from a chimney indicates that plumes and jets are bent over by a cross wind and rise at a rate which diminishes with distance until the plume is effectively horizontal. Thus for distances downwind of the maximum rise, concentrations may be estimated from the formulae of the preceding section by considering the source to be at an effective height greater than the actual height by an amount equal to the plume rise,  $h_p$ . For points nearer the source, the concentration may be estimated by considering the spread about the plume centre-line to be the same as in the absence of buoyancy or momentum.

In both the above cases it is necessary to estimate the plume trajectory. Two methods for doing this are given below and the results compared with observed plume behaviour.

If vertical velocity  $w$  decays with distance  $s$  along the plume centre-line according to the law  $w = A/s^p$ , where  $A$  is constant for a given plume, then the distance at which the plume inclination to the horizontal is  $\alpha$ , given by:

$$s = \left( \frac{A}{\bar{u}} \cot \alpha \right)^{1/p}$$

For a jet it follows from conservation of momentum that  $p = 1$ , hence the co-ordinates of the trajectory are:

$$x = \frac{A_m}{\bar{u}} (\operatorname{cosec} \alpha - 1) \quad (9)$$

$$z = \frac{A_m}{\bar{u}} \ln (\cot \alpha + \operatorname{cosec} \alpha) \quad (10)$$

A suitable value for  $A_m$  taken from Squire<sup>15</sup> is:

$$A_m = 6.5 v_0 d \left( \frac{\rho_0}{\rho_a} \right)^{\frac{1}{2}} \quad (11)$$

\* For further discussion see the Appendix.

Similarly for a buoyant plume, it follows from the conservation of buoyancy and the known decay of concentration, that  $\rho = 1/3$ . Thus the centre-line coordinates are:

$$x = \left(\frac{A_b}{\bar{u}}\right)^3 (2 + \operatorname{cosec}^3 \alpha - 3 \operatorname{cosec} \alpha) \quad (12)$$

$$z = \frac{3}{2} \left(\frac{A_b}{\bar{u}}\right)^3 (\cot \alpha \operatorname{cosec} \alpha - \ln(\cot \alpha + \operatorname{cosec} \alpha)) \quad (13)$$

Equation (13) was first published by Sutton.<sup>19</sup> From the work of Yih<sup>17</sup>  $A_b$  is given by:

$$A_b = 4.7 \left(\frac{Q\Delta\rho_0 g}{\rho_a}\right)^{1/3} \quad (14)$$

To facilitate the use of equations (9), (10), (12), and (13) above, corresponding values of

$$\alpha, x\left(\frac{\bar{u}}{A_m}\right), z\left(\frac{\bar{u}}{A_m}\right), x\left(\frac{\bar{u}}{A_b}\right), \text{ and } z\left(\frac{\bar{u}}{A_b}\right)$$

are given in Table IV.

TABLE IV.—Trajectories of Jet and Plume

$\alpha$ (°)	Jet		Plume	
	$x\left(\frac{\bar{u}}{A_m}\right)$	$z\left(\frac{\bar{u}}{A_m}\right)$	$x\left(\frac{\bar{u}}{A_b}\right)^3$	$z\left(\frac{\bar{u}}{A_b}\right)^3$
85	0.0038	0.087	0.000044	0.00067
80	0.0154	0.175	0.000718	0.00544
75	0.0353	0.265	0.00378	0.0188
70	0.0642	0.356	0.0126	0.0464
65	0.103	0.451	0.0332	0.0955
60	0.155	0.549	0.0755	0.176
55	0.221	0.653	0.157	0.303
50	0.305	0.763	0.308	0.499
45	0.414	0.881	0.586	0.799
40	0.556	1.01	1.10	1.26
35	0.743	1.15	2.07	2.00
30	1.000	1.32	4.00	3.22
25	1.37	1.51	8.15	5.35
20	1.92	1.74	18.2	9.45
15	2.86	2.03	48.1	18.6
10	4.76	2.44	176	45.3

It will be seen that both sets of formulae given above predict a continued rise of the plume. This is a consequence of the assumed conservation of momentum or buoyancy, and will not be true at large distances from the source. To overcome this difficulty, Sutton suggested taking the maximum rise as the value of  $z$  for which  $\alpha = 10^\circ$ . Thus the corresponding values for plume rise are:

$$\text{momentum jet: } h_r = 2.44 \frac{A_m}{\bar{u}}$$

$$\text{buoyant plume: } h_r = 45.3 \left(\frac{A_b}{\bar{u}}\right)^3$$

In addition the assumed vertical velocity at the centre-line is an over-estimate, so the plume rise may also be expected to be too high. This may be useful since it sets an upper limit to the trajectory.

An alternative approach has been suggested by Bosanquet<sup>22</sup> who evaluated the vertical velocity at the centre-line as a function of time by an involved route based on the observed radial dilution of plumes and jets. This rise after time  $t$  is then given by:

$$h_r(t) = \int_0^t w dt.$$

Bosanquet suggested the maximum rise occurs after 200 s, and obtained a numerical solution for plume rise due to momentum and buoyancy alone and in combination. Formulae and tables of functions from which the trajectory may be calculated are given in the original reference.

Observed plume trajectories reported in the literature by Bosanquet *et al.*,<sup>23</sup> Csanady,<sup>24</sup> Moses and Strom,<sup>25</sup> and Stewart, *et al.*,<sup>4</sup> have been compared with the predictions of equations (9)–(14) and of Bosanquet. This comparison shows that on average, Bosanquet's treatment gives the better agreement with observation but that it usually underestimates the rise except near to the source. Equations (9)–(14) always over-estimate the rise.

## Practical Application

### General discussion

The formulae presented in the foregoing sections may be used to estimate the distribution of effluent round a source which has small dimensions in comparison with the distances involved. From the complexity of the problem it is unlikely that they will predict exactly the concentration at any point, but it is possible to calculate a maximum value which will not be exceeded. Conversely, it is possible to predict the maximum distance required to dilute a given discharge to a safe concentration.

The simplest calculation involves the jet formula. This gives minimum dilution, or maximum extension of hazard area. Hence the instantaneous concentration at a distance  $x$  from the source will not exceed:

$$9c_0 \frac{d_0}{x} \left(\frac{\rho_a}{\rho_0}\right)^{\frac{1}{2}}$$

and the time mean concentration will not exceed:

$$6c_0 \frac{d_0}{x} \left(\frac{\rho_a}{\rho_0}\right)^{\frac{1}{2}}$$

Thus if  $c_s$  is the limiting safe concentration, the range of hazard will be:

$$9 \frac{c_0}{c_s} d_0 \left(\frac{\rho_a}{\rho_0}\right)^{\frac{1}{2}}$$

or:

$$6 \frac{c_0}{c_s} d_0 \left(\frac{\rho_a}{\rho_0}\right)^{\frac{1}{2}}$$

depending on the type of concentration appropriate to the hazard.

Because of the conservative nature of the jet formula, the range calculated from it may be unrealistic. In this case one of the other formulae might be applied. For instance with buoyant effluents minimum dilution occurs in a quiescent atmosphere. Thus the quiescent atmosphere formula (equation (4)) should be safe for predicting vertical range of hazard. In the presence of a cross wind the range is further reduced by two factors: the curvature of the plume axis and dilution by eddies in the wind. The former is relatively unimportant, while the latter is a major effect. When calculating the trajectory in a cross wind the method chosen should be conservative for the hazard assessed.

### Numerical example

A simple flammability problem will now be illustrated by a numerical example. This will also serve to show the relative effect of factors involved in dilution problems. Consider a mixture of a heavy organic vapour and air containing 30% vapour discharged above a flat roof at a rate of 10 ft<sup>3</sup>/min from a vertical pipe 1 in. i.d. The specific gravity of vapour

relative to air is 3 and the lower limit of inflammability is 1% by volume. It is required to specify the height of pipe necessary to avoid a flammable concentration at a height of 2 ft above the roof level.

#### APPLICATION OF JET FORMULA

The worst case would occur if the pipe terminated in a U-bend and the jet were directed towards the roof in still air. Applying formula (3) with  $k_2 = 9$ , the height is found as follows:

$$\begin{aligned} \text{height required} &= 2 + 9 \times 30 \times \frac{1}{1.6} \left( \frac{1}{1.6} \right)^{\frac{3}{2}} \\ &= 20 \text{ ft approx.} \end{aligned}$$

This over-estimates height because in this example the jet would degenerate to a plume before reaching the roof level.

#### APPLICATION OF PLUME FORMULA

If the pipe ended in a U-bend or in a 90° elbow, dilution in still air would proceed initially as for a jet and later the plume would fall under gravity. Jet dilution persists for about 2.3  $F^{\frac{1}{2}}$  diameters, which in this case is approximately 1.2 ft. Over this distance the average concentration falls to less than half the initial value, but since the buoyancy flux is unaltered, this is without effect on the height required for buoyant dilution. Thus to estimate height, jet dilution may be neglected and it is sufficient to apply equation (4) to the initial conditions with  $k_4 = 17$ .

$$\begin{aligned} \text{Hence height required} &= 2 + \left( \frac{(17 \times 30)^3}{6^2 \times 0.6 \times 32.2} \right)^{1/5} \\ &= 13.4 \text{ ft (approx.)} \end{aligned}$$

Both the foregoing calculations have been for still air. In the presence of a cross wind the required dilution occurs through a smaller height because of two factors:

- the bending of the plume axis,
- the diluting effect of eddies present in the wind.

Separate consideration of these effects follows.

#### CURVATURE OF PLUME AXIS

From the previous section the required dilution occurs when the plume trajectory is 11.4 ft. For a buoyant plume in a cross-wind  $\bar{u}$ , the length of trajectory is given by:

$$s = \left( \frac{A_b}{\bar{u}} \cot \alpha \right)^3$$

In the present case  $A_b = 6.94 \text{ ft}^4/\text{s}^{-1}$ , so the inclination of the trajectory to the horizontal after 11.4 ft is  $\cot^{-1} 0.324 \bar{u}$ . From Table IV the heights corresponding to different wind-speeds may be computed as shown in Table V.

TABLE V.—Computed Heights for Various Windspeeds

Windspeed $\bar{u}$ (ft/s)	$\cot \alpha$	$\alpha$ (°)	$z$ (ft)	Height of pipe (ft)
0	0	90	11.4	13.4
1	0.324	72	11.0	13
5	1.62	32	7.0	9
10	3.24	17	4.8	7
20	6.48	say 10	1.9	4

#### DILUTION BY EDDIES IN WIND

Assuming neutral conditions, the distance downwind at which dilution to the lower limit occurs may be found from equation (8), neglecting the exponential term, as follows:

$$x \approx \left( \frac{3}{60 \times \pi \times 0.24 \times 0.14 \times 0.01 \times \bar{u}} \right)^{0.571} = 9.05 \left( \frac{1}{\bar{u}} \right)^{0.571}$$

Thus  $x$  depends on windspeed. For each value of  $x$  a corresponding value of  $z$  may be found from Table IV. A series of results calculated in this way by slide rule is shown in Table VI. These calculations strictly refer to the three

TABLE VI.—Distance at which Dilution Occurs

Windspeed $\bar{u}$ (ft/s)	$x$ (ft)	$x \left( \frac{\bar{u}}{A_3} \right)^3$	$z \left( \frac{\bar{u}}{A_3} \right)^3$	$z$ (ft)	height = $z+2$ (ft)
0.5	13.45	0.0402	0.110	36.7	39
1	9.05	0.216	0.383	16.0	18
2.5	5.36	2.02	1.97	5.25	7.3
5	3.60	10.8	6.53	2.18	4.2
10	2.42	57.9	21.1	0.88	2.9
20	1.63	312	45.3	0.24	2.2

minutes time-mean concentration, but in view of Katan's work are probably adequate. If the instantaneous concentration is considered to be double the time-mean, the tabulated results will apply at twice the wind speed shown.

#### COMPARISON OF RESULTS

To facilitate comparison, the results of the above calculations are summarised below.

System	Height of pipe (ft)
Jet dilution—still air	20
Plume dilution—still air	13.4
Plume dilution—displaced by wind	
windspeed 1 ft/s	13
5 ft/s	9
Dilution by eddies in wind	
windspeed 1 ft/s	18
5 ft/s	4.2
20 ft/s	2.2

It will be seen that the height specified by the jet formula is unnecessarily large. However there can be no doubt that it would be safe. The result for a falling plume in still air corresponds to the worst dilution likely to be encountered in practice and may also be considered safe. However if emission could be restricted to occasions of high windspeed, a much shorter pipe would suffice. In this case, because of initial momentum and buoyancy, dilution should exceed that due to eddies in the wind alone. Hence figures based on such dilution should be safe for the wind-speeds specified. At low wind-speeds the principal effect of wind is to increase the trajectory for a given fall, and a safe value should result from application of the plume dilution formula to a suitably curved trajectory. In the example considered this gives a lower height than Sutton's formula for winds of 1 ft/s or less. However, this situation is relatively unimportant and is barely distinguishable from still conditions. The most noticeable feature of the calculations is that dilution occurs more rapidly under the influence of the eddies already present in the atmosphere than under those generated by the relative motion of effluent and atmosphere.

#### Environmental effects

With the exception of the Sutton formula the results so far presented apply strictly to isolated sources only. In the proximity of solid objects some modification may be required. Generally solid boundaries influence the concentration distribution in two ways: by reflection of the effluent and by modifying the aerodynamic flow pattern.

Reflection may be simulated (as in the Sutton formula) by the method of images. This introduces virtual sources in such positions that when compounded in space with the actual source, they produce surfaces of no net flux in the same position as the actual surfaces.

Aerodynamic effects of importance are layering and wake development. The former occurs when the turbulent stresses are insufficient to overcome the buoyancy forces in a layer

adjacent to a boundary. In practice it may be avoided by securing adequate dilution before the plume reaches the boundary. This aspect of mixing has been extensively studied by Bakke and Leach.<sup>26</sup>

The effects of wakes of buildings on plumes has been discussed in connection with atmospheric pollution by Scorer and Barratt<sup>27</sup> who have proposed simple methods for calculating concentrations. The main effect of wake seems to be displacement of the plume, so although the plume may follow an unexpected course, the concentration at any point will not exceed the maximum possible for that distance from the source.

From the above discussion it would appear that the presence of buildings and other obstacles need not preclude the application of the ideal formulae appropriate to isolated sources.

#### Extension to dusts

The treatment given for gases and vapours may be extended to dusts by superimposing the appropriate gravitational fall on the turbulent concentration distribution. One way of doing this has been described by Bosanquet, Carey, and Halton.<sup>23</sup>

### Conclusions

In view of the complexity of the problem of atmospheric diffusion no simple treatment can accurately predict the distribution of effluent from a vent.

The most extensively studied turbulent dilution system is the momentum jet discharging into a still environment. The agreement between different investigators suggests that concentration in such jets can be estimated with fair reliability. It is certainly possible to predict a maximum concentration which will not be exceeded at a given distance from the source.

Since dilution by other modes available in atmospheric diffusion is more rapid than for a momentum jet, concentrations appropriate to the latter are safe in predicting the extent of hazard areas.

In some cases a less conservative approach is possible, employing other methods of calculation, such as dilution of a buoyant plume or dilution by eddies in the atmosphere. These situations have been less investigated, and the results are therefore not so well established as those for a jet.

There is need for further experimental work to place the definition of hazard areas on a rational basis. If this paper stimulates this it will achieve a worthwhile end.

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### Symbols Used

$A_b$  = a constant for a buoyant plume defined by equation (14).

$A_m$  = a constant for a momentum jet defined by equation (11).

$c_0$  = initial concentration of emitted substance.

$c_s$  = safe concentration of emitted substance.

$\bar{c}_x$  = average time-mean concentration in jet at a distance  $x$  from the virtual source.

$\bar{c}_{xr}$  = time-mean concentration at a point axial distance  $x$  and radial distance  $r$  from the virtual point source.

$\bar{c}(xyz)$  = time-mean concentration at point  $xyz$ .

$C_y$  = generalised diffusion coefficient in  $y$  direction (Sutton).

$C_z$  = generalised diffusion coefficient in  $z$  direction (Sutton).

$d_o$  = diameter of orifice.

$F$  = modified Froude number.

$g$  = acceleration due to gravity.

$h$  = height of source above ground.

$h_r$  = plume rise above source level.

$k$  = constants of proportionality; different numerical subscripts for different constants.

$m_o$  = mass flow rate of gas at orifice.

$m_x$  = mass flow rate of gas at distance  $x$  from virtual source.

$n$  = meteorological parameter relating to the transport properties of the atmosphere.

$p$  = index in the law of velocity decay along the centre line.

$Q$  = strength of source of emission.

$Q_F$  = volumetric filling rate of fuel tank.

$r$  = radial co-ordinate in a symmetrical jet or plume.

$s$  = distance along the centre line of a deflected plume.

$t$  = time.

$\bar{u}$  = mean wind speed.

$v_0$  = initial velocity of jet.

$w$  = vertical velocity of jet or plume.

$x$  = downstream or downwind distance from a virtual point source.

$y$  = cross wind horizontal co-ordinate; origin at virtual point source.

$z$  = vertical co-ordinate; origin at virtual point source.

$\alpha$  = inclination of plume to horizontal.

$u$  = viscosity.

$\rho_a$  = density of ambient atmosphere.

$\rho_0$  = density of issuing gas.

$\bar{\rho}_x$  = average time-mean density at  $x$ .

$\bar{\rho}_{x0}$  = time-mean density on centre line at  $x$ .

$\Delta\rho_0$  = initial density difference between plume and atmosphere.

$\sigma_y$  = standard deviation of Gaussian concentration profile in  $y$  direction.

$\sigma_z$  = standard deviation of Gaussian concentration profile in  $z$  direction.

### APPENDIX

#### Is Sutton's Treatment Applicable to Short Range Effects?

After assessment of this paper by referees the above question was asked and justification required for the application of Sutton's method to short range problems.

The use of Sutton's treatment near to the source of emission is permissible for calculating the extent of hazard areas if:

(a) experimental surveys indicate that it does not underestimate concentration, or

(b) the assumptions on which it depends can be shown to obtain, or

(c) the assumptions, while not proved, seem reasonable.

Clearly in the last case the results must be treated with some reservation, but even so they have practical usefulness. Separate consideration will now be given to experimental results and assumptions.



### Experimental results

The only survey known to the author in which concentrations were measured close to a source is the work of Katan previously mentioned.<sup>1</sup> Katan measured almost instantaneous concentrations at distances ranging from 6 in. to 9 ft downwind of a filling point (or vent) in aircraft fuel tanks. He presented his results as the product of maximum concentration and windspeed divided by the volumetric filling rate,  $Q_F$ . The maximum value of this concentration parameter at a given distance for the complete series of experiments is plotted (with changed units) against distance in Fig. 3. Also for comparison a solid line is shown for the value of the parameter calculated from equation (8) for a three-minute mean sampling time under neutral conditions assuming  $h$  equal to zero and the effluent to be saturated with petrol vapour at 20°C (*i.e.* partial pressure approximately 2/7 atm).

It will be seen that the Sutton relation overestimates concentration.

### Validity of assumptions

The assumptions on which Sutton's treatment is based are:

- (1) conservation of mass,
- (2) perfect reflection by the ground,
- (3) constant velocity across the plume,
- (4) Gaussian concentration profile,
- (5) the spread of the plume (or the standard deviation of concentration profile) is given by Sutton's coefficients.

The first assumption presents no difficulty. The second gives rise to the exponential term of equation (8) and is generally insignificant near a source where reflection has not yet occurred. In any case retention of a flammable vapour by the ground or other boundary is likely to be small. The third assumption is substantially correct if the plume is narrow and initial momentum is dissipated quickly. The fourth assumption is approximately true of all diffusing systems investigated in the field or laboratory. The fifth assumption is the crucial test of applicability provided the initial momentum is negligible. Sutton's treatment predicts a roughly conical spread of the effluent. If the boundary is arbitrarily defined as occurring where the concentration has fallen to one-tenth of the maximum value at the same downwind distance from the source, then according to Sutton's equations applied over the first 10 ft from the source under neutral conditions, the average semi-angular spreads in the vertical and horizontal directions are about 10° and 16° respectively. These values seem quite reasonable and may

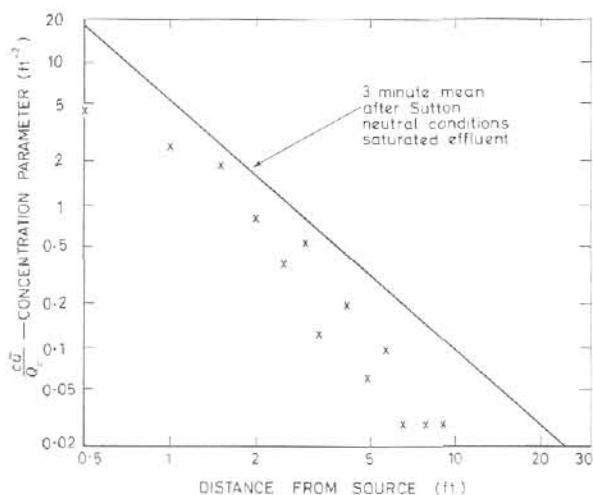


Fig. 3.—Maximum values of concentration parameter plotted against distance

be compared with the symmetrical spread of jets and plumes discharging into quiescent atmospheres for which the semi-angles are found to be in the range 9° to 16°. Hence if a jet or plume does not appreciably change its angular spread on degenerating to a drifting cloud it would appear that Sutton's values are approximately correct. Observation of the spread of jets of steam or smoke may thus afford guidance in assessing the validity of the treatment.

### Conclusion

From the above considerations it is concluded that Sutton's treatment may be applied with discretion in short range problems in the absence of a better method of calculation. Any criticism of its inclusion here should take account of the conclusions to the main paper.

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