THE MODIFIED CUMULATIVE FREQUENCY EQUIVALENT

A POTENTIAL MEASURE OF OFF-SITE RISK

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A numerical methodology has been developed to provide a measure of local off-site risk from chemical hazardous installations that could be used for the purposes of land use planning. Called the Modified Cumulative Frequency Equivalent (MCFE) it is a measure of local risk that is directly comparable with F-N criterion lines previously described by the Health and Safety Executive (HSE). MCFE incorporates a measure of scale aversion consistent with that previously used by HSE in the context of land use planning. An MCFE may be calculated for a particular installation or for a defined population subject to risks from a number of installations. The use of MCFE would be in addition to existing land use planning controls based on individual risk calculations. MCFE allows advice to be given concerning single large development proposals, possibly beyond the current consultation limits, and can also be used to consider the effect of the accumulation of smaller developments in the vicinity of any installation over a period of time.

This paper describes the mathematical basis for the MCFE method. An example of the use of MCFE to describe the off-site risk associated with a chemical Hazardous Installation is given. The example also describes the use of MCFE to advise on proposed developments in the vicinity of the installation.

KEYWORDS: Hazardous Installations, off-site risk, land use planning, numerical methodology

DESCRIPTION

Cumulative frequency curves can be used to describe local risk from major accident establishments. This method is based on the use of standardised cumulative frequency curves with known properties to derive relatively simple formulae, incorporating decision criteria, for possible use in a land use planning context.

The Expectation Value (EV), in the context of this method, is the number of fatalities per million per year that may be expected from specified activities. The numbers of persons affected in any one incident is not distinguished. The frequency of an outcome is given the same weighting as the consequences. It is therefore aversion neutral. Thus a

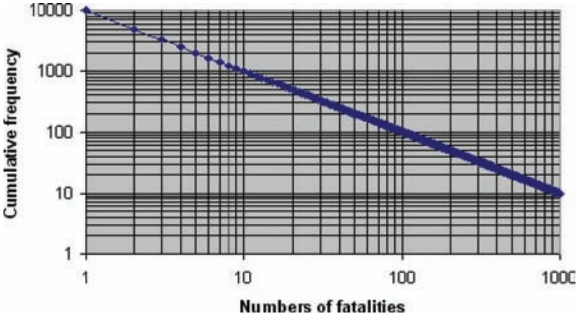

Figure 1. F-N criterion line for local risk

single accident affecting 10 persons at a frequency of 1 in 1,000 per annum is considered equivalent to a single accident affecting 100 persons at a frequency of 1 in 10,000 per annum. Both would have the same expectation value of 10,000.

The MCFE is a new measure. The MCFE would be used in addition to the existing individual risk based methodology which sets the consultation distance and three zones around each installation for the purposes of land use planning control (HSE 1989).

The MCFE is calculated using a formula that includes the EV and the maximum number of persons who may be affected at once (Nmax). The MCFE is directly comparable with the well-known criterion point for local societal risk extrapolated as a line with slope -1 (Figure 1).

This line passes through the point $N = 50$, $F = 200$ (chances per million per year risk of death 'cpm'), HSE's basic criterion for the limit of tolerability (HSE 2001).

Any F-N plot for an installation can be compared with this point, and should the value of the cumulative frequency be exceeded then the risk is determined to be excessive. The criterion can be extrapolated to any other point on the line. However, when incorporating scale aversion into the decision making process, it is necessary to ensure that the comparison at 50 fatalities is not affected.

A further line could be drawn below this line which passes through the point $N = 50$, $F = 2$ cpm. This could be considered to be the line representing broadly acceptable risk (Figure 2).

These lines are not comparable with the EV as the EV is an integrated measure of risk (otherwise called a risk integral). However it can be shown that the EV is the area under the line (for a linear-linear plot) between any two points. It can also be shown that the EV for the sector of the line between 1 and 10 is equal to the EV for the sector of the line between 10 and 100, and similarly for the sector between 100 and 1000 (Figure 3).

In theory the EV could increase to any value as Nmax increases, and providing the F-N curve does not cross the line, the result would not exceed the criterion.

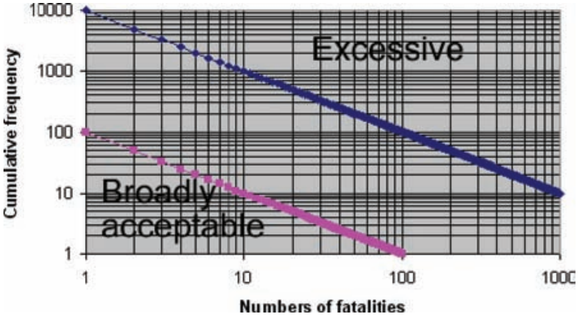

Figure 2. F-N criteria lines for local risk

SCALE AVERSION

Traditionally in HSE, measures of risk for land use planning in the vicinity of major chemical installations have included a high degree of scale aversion. This was introduced into the formulae many years ago (Cassidy 1996) when HSE's views on societal concerns in general had not been formulated. This degree of aversion has been maintained in the MCFE methodology.

THE SIZE OF THE INSTALLATION

If we consider a set of chlorine installations consisting of two vessels and associated activities, but of various vessel sizes so that the total quantity ranges from 50te to 200te with a proportionate number of road tanker deliveries, then for a given population in

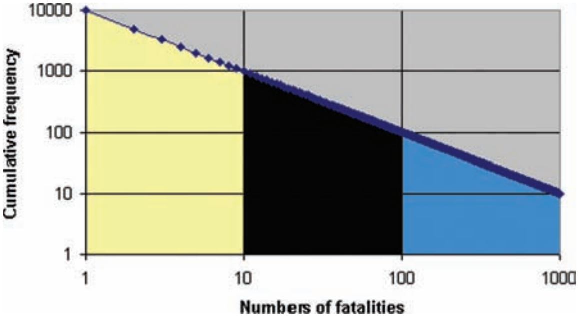

Figure 3. Areas of equal expectation value

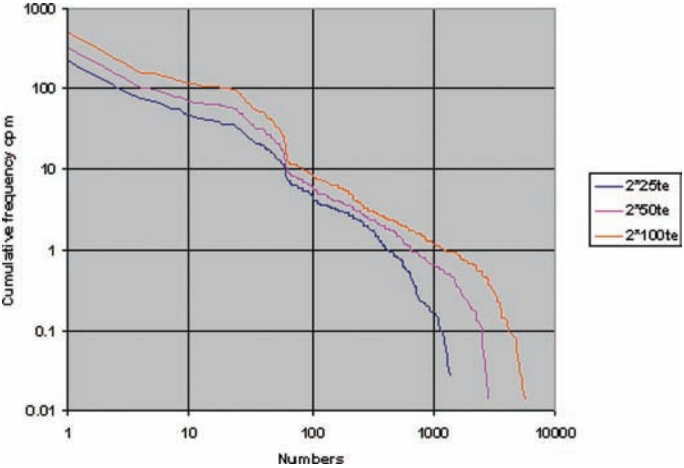

Figure 4. F-N curves for chlorine installations with two vessels

the vicinity we might produce a set of F-N curves and use the appropriate curve as the basis of decision making (Figure 4).

In practice the full F-N curve is difficult to calculate robustly due to uncertainty and sensitivity. A wide range of results are possible depending on the depth of analysis and the assumptions made. None of these results can claim to be 'correct'. The resulting curve is often not smooth and therefore difficult to compare with a criterion line.

Traditionally HSE has not carried out F-N analysis for hazardous chemical installations.

To avoid these problems the MCFE method only requires estimates of EV and Nmax.

EXAMPLE

As an example, the 100te chlorine installation consisting of two 50te storage vessels with one road tanker delivery per week is considered. The residential population data is shown in diagrammatic form (Figure 5).

In this case the risk is described by a contour plot of individual risk and the results may be combined with population data (Figure 6) and used to calculate the EV. When this is done it is correct to estimate the mean risk, rather than the median risk in each population data square. This is more important close to the source, and in particular for the source square itself where there is most deviation between the two values. The use of greater definition close to the source may facilitate this.

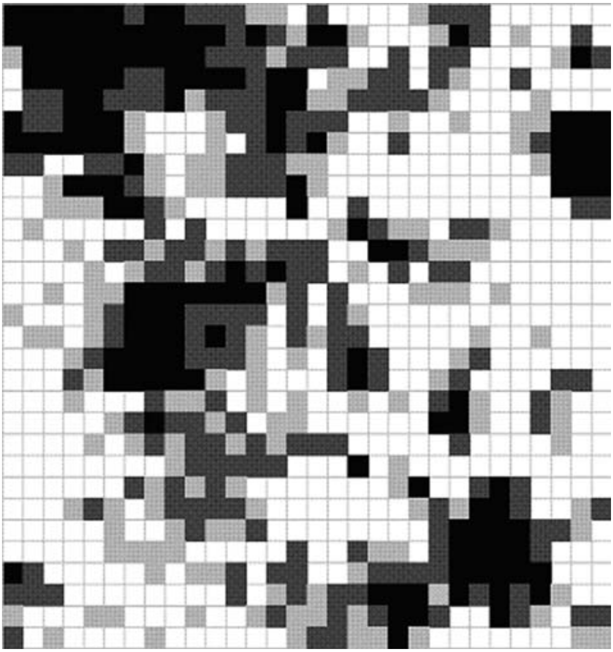

Figure 5. Population data on a 30 km × 30 km grid (black >1000 per km^2 to white <50 per km^2)

ESTIMATION OF NMAX

For the example Nmax may be calculated by describing the 'worst case' toxic event. The selection of the 'worst case' event is the same as used in other HSE calculations (Hirst 2000). For this case the consequences are represented by six co-axial zones comprising three for indoor exposure and three for outdoor exposure obtained from an appropriate toxic gas dispersion model (Figure 7).

The three zones correspond to:

- An inner zone at the SLOD (LD50 or greater) within which 75% fatalities are assumed.
- A middle zone (between LD50 and LD10) within which 30% fatalities are assumed.
- An outer zone (between the 'Dangerous Dose' LD01 and LD10) within which 5.5% fatalities are assumed.

The predicted number of fatalities from each of the 6 zones are summed to give a total. The number resulting from orientation to the worst direction is Nmax. For the example this number was 2573.

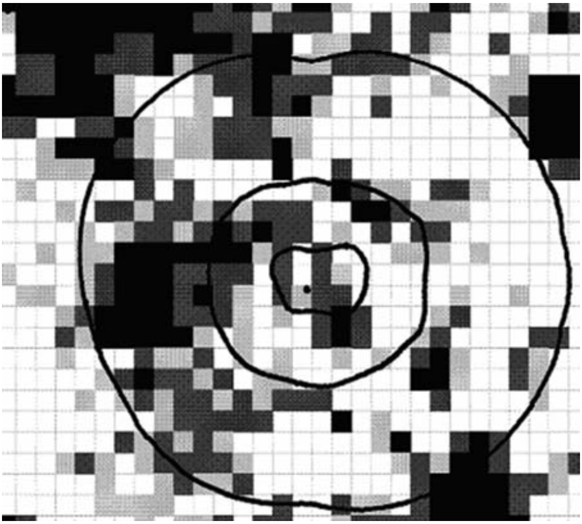

Figure 6. Individual risk contours (0.1, 0.001, 0.00001 cpm) superimposed on the grid of population

The MCFE method uses this information to derive a standardised f-n data set. The f-n pairs are then used to calculate a cumulative frequency equivalent curve (Figure 8).

This cumulative frequency equivalent curve is then enhanced to incorporate scale aversion to the same degree as previous HSE methodology (Carter 1995). This is achieved by multiplying each value of the cumulative frequency by the corresponding number N. This modified F-N curve is then normalised by dividing each value of the cumulative frequency by 50 so that the value at 50 fatalities is unchanged (Figure 9).

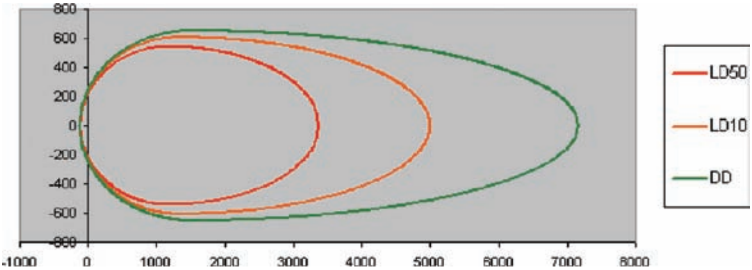

Figure 7. 50te Chlorine release stable weather indoor dose (downwind and crosswind metres)

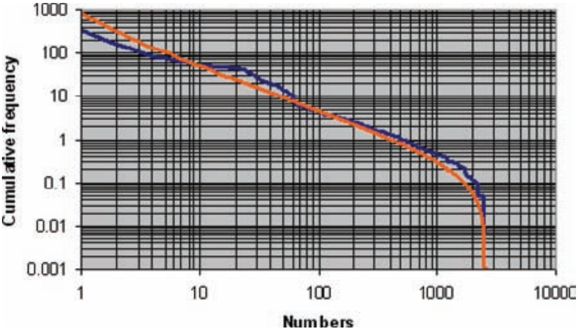

Figure 8. Cumulative frequency equivalent and actual F-N curves

This allows comparison with the criterion F-N line described above. The maximum ratio of the cumulative frequency at any point on the curve divided by the equivalent point on the criterion F-N line, gives the required result.

MCFE RATIO

Simple formulae have been derived which preclude the need for lengthy calculations. (Appendix 1).

There are two formulae, one for unidirectional hazards such as toxic gas clouds dispersing over the land, and one for omnidirectional hazards such as fireballs and vapour cloud explosions. This is similar to previous HSE methodology (Hirst 2000).

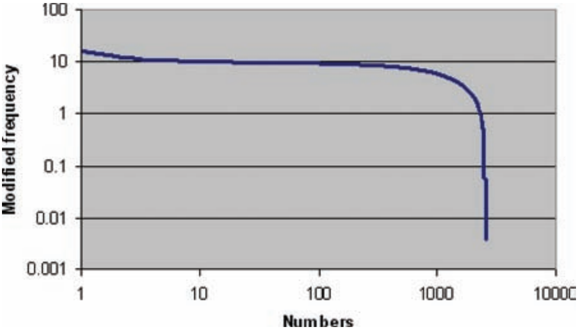

Figure 9. Modified cumulative frequency equivalent curve

Unidirectional

$$\mathrm{MCFE}_{\mathrm{ratio}} = [\mathrm{EV} * \mathrm{Nmax}]/[2 \times 10^6 * (0.577 + \mathrm{Ln}(\mathrm{Nmax}))] \tag{1}$$

Omnidirectional

$$\mathrm{MCFE}_{\mathrm{ratio}} = [\mathrm{EV} * \mathrm{Nmax}]/[5 \times 10^5 * (0.577 + \mathrm{Ln}(\mathrm{Nmax}))] \tag{2}$$

Thus if the $\mathrm{MCFE}_{\mathrm{ratio}}$ exceeds 1 then the risk would exceed the criterion, and if the $\mathrm{MCFE}_{\mathrm{ratio}}$ were less than 0.01 then the risk would be considered broadly acceptable.

EXAMPLE

For the example installation the EV is 5221 and Nmax is 2573. In this case 89% of the EV would be produced by elements within the likely consultation distance and 98% within twice the likely consultation distance.

The MCFE ratio using equation 1 is 0.80. This does not exceed the upper criterion but may result in scrutiny of planning proposals that could significantly increase the EV or Nmax, including the accumulation of developments over time.

For example a large residential population increase of 4000 persons may be proposed at a location to the south east of the installation at an average distance of 5.5 km. This would be well beyond the distance that development would otherwise be controlled.

The EV would increase to 5274 and Nmax to 2803. This would increase the MCFE ratio to 0.87. The new value of the MCFE ratio does not exceed the upper criterion and the increase is less than 10%. This may be considered acceptable from a safety viewpoint, depending on the development control policies that may be in operation.

CONCLUSION

The MCFE is a simple measure of off-site risk which includes scale aversion, and can be compared with a standard criterion line. Results are dependent upon the type and size of installation and the size and distribution of the local population.

The ratio of the MCFE with the criterion line can be calculated by a simple formula which avoids the need for F-N diagrams.

The value of the ratio can be used as a basis for decision making in land use planning.

ACKNOWLEDGEMENTS

The author would like to thank colleagues in the HSE, for their assistance in the preparation of this paper.

Nothing in this paper should be interpreted as a statement of HSE policy.

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APPENDIX 1

DERIVATION OF THE MCFE FORMULAE

It is required to derive formulae for the MCFE ratio based on the expectation value (EV) and Nmax using previously defined normalised relationships between numbers and frequency.

The EV is defined as:

$$\mathrm{EV} = \Sigma(\mathrm{f} \times \mathrm{n}) = \Sigma(F) \tag{1}$$

where f is the frequency and n is the number of fatalities in each f-n pair, and F is the cumulative frequency at each value of N (n or more).

For the **unidirectional** case the underlying relationship is defined as:

$$\mathrm{f} = \mathrm{A}/\mathrm{n}^2 \tag{2}$$

$$\therefore\ \mathrm{EV} = \Sigma(\mathrm{A}/\mathrm{n}) = \mathrm{A} \times \Sigma(1/\mathrm{n}) \tag{3}$$

$$\therefore\ \mathrm{A} = \mathrm{EV}/\Sigma(1/\mathrm{n}) \tag{4}$$

$\Sigma(1/n)$ is a well known infinite series with an approximate value of $0.577 + \mathrm{Ln(Nmax)}$ (Figure 10). The approximation has an error of less than 1% when Nmax is greater than 14.

This gives the final form of the expression for A which could be used to determine f for each value of n:

$$\mathrm{A} = \mathrm{EV}/(0.577 + \mathrm{Ln(Nmax)}) \tag{5}$$

The cumulative frequency at each value of N is easily determined (Figure 11).

To incorporate aversion, the standard form of the LUP Risk Integral is used:

$$\mathrm{RI}_{\mathrm{LUP}} = \Sigma(\mathrm{F} \times \mathrm{N}) \tag{6}$$

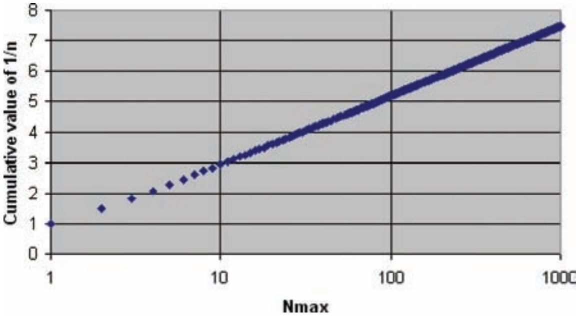

Figure 10. Σ (1/n) as a function of Nmax

To enable comparison with the criterion line, and ensure the comparison value for 50 fatalities remains valid, each cumulative frequency value F is multiplied by N/50 to give modified cumulative frequency (MCF) values (Figure 12).

The value of the MCFE can be divided by the equivalent value of F from the criteria line at every point. The maximum value of this ratio is called the MCFE ratio.

It can be shown (the proof is not included here) that this maximum value always occurs at a value of N equal to Nmax/2. As an example the graph for EV = 100,000 and Nmax 1,000 is shown (Figure 13).

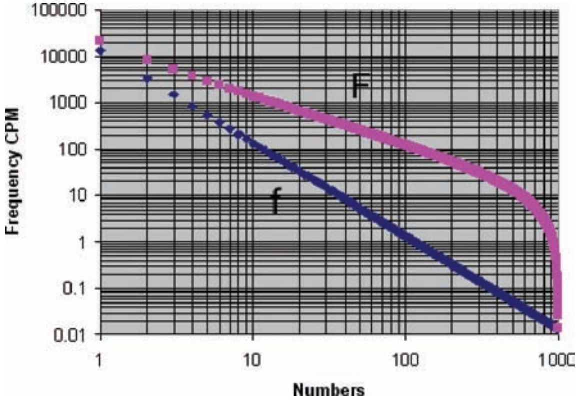

Figure 11. Frequency(f) and Cumulative Frequency (F)

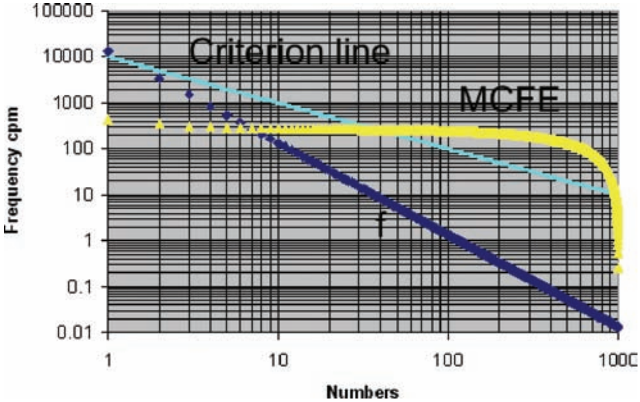

Figure 12. Frequency(f) and modified cumulative frequency equivalent (MCFE) for a unidirectional event with criterion line

Also at this point it can be shown (the proof is not included here) that the value of the cumulative frequency is given by a simple formula:

$$F_{Nmax/2} = A/Nmax \tag{7}$$

The value of the MCFE at this point is therefore:

$$\begin{aligned} MCFE &= (A/Nmax) \times Nmax/(2 \times 50) \\ &= A/100 \\ &= EV/(0.577 + Ln(Nmax))/100 \end{aligned} \tag{8}$$

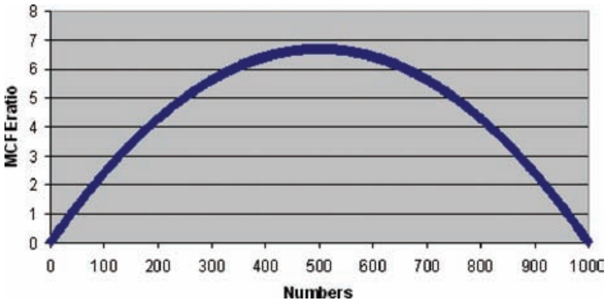

Figure 13. Unidirectional modified cumulative frequency (MCFE) ratio maximum point

The value of F from the criteria line at Nmax/2 is:

$$F_{criteria} = 10000 \times 2/Nmax \tag{9}$$

Dividing (8) by (9) gives the MCFE ratio:

$$MCFE_{ratio} = EV \times Nmax/(0.577 + Ln(Nmax))/2E6 \tag{10}$$

For the **omnidirectional** case the underlying relationship is defined as:

$$F = B/n \tag{11}$$

$$\therefore EV = \Sigma(B/n) = B \times \Sigma(1/n) \tag{12}$$

$$\therefore B = EV/\Sigma(1/n) \tag{13}$$

As noted above $\Sigma(1/n)$ is a well known infinite series with an approximate value of $0.577 + Ln(Nmax)$.

$$\therefore B = EV/(0.577 + Ln(Nmax)) \tag{14}$$

The MCFE for each value of N is F × N/50.

It is clearly shown (Figure 14) that the maximum ratio compared to the criteria line is at Nmax.

The value of MCFE at this point is therefore:

$$MCFE_{Nmax} = B/Nmax \times Nmax/50 = EV/(0.577 + Ln(Nmax))/50 \tag{15}$$

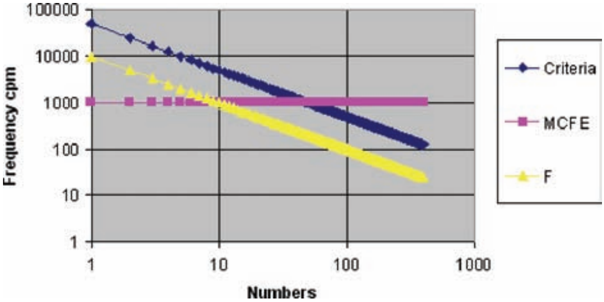

Figure 14. Cumulative frequency (F) and modified cumulative frequency equivalent (MCFE) for an omnidirectional event with criteria line

The value for the criteria at Nmax is:

$$F_{criteria} = 10000/Nmax \tag{16}$$

Dividing 15 by 16 gives:

$$MCFE_{ratio} = EV \times Nmax/(0.577 + Ln(Nmax))/5E5 \tag{17}$$